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A General Existence Principle for Fixed Point Theorems in One Parameter Case of Soft D_a-Metric Space

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Abstract In this paper, we take the case of soft points into consideration and propose a new metric structure called soft D_a-metric space for a specific orbit defined with soft points. In order to establish fixed point results in the modified metric space, we modify a few existing definitions in the sense of soft points.

Keywords soft set; soft metric; D-metric; soft D-metric; fixed point theorem

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1. Introduction

The study of various generalisations in metric spaces is a pioneering work in the field of mathematical research dealing with ambiguity problems. The main motivation for obtaining these generalizations is because a broad classification of metric spaces provides a powerful tool for tackling issues involving uncertainty in numerous disciplines of mathematics, such as approximation theory and variational inequalities. In multidisciplinary areas of science, various generalizations of metric spaces have a wide range of applications. Problems involving finding solutions will constantly be dependent on fixed point theory because it ensures that solutions exist. Fixed point theory has application related to electrical engineering, where there is a problem concerning the transformation of solar energy to electric power which can be seen in [1]. Dhage [2] introduced the concept of D-metrics and developed various properties and results in their later attempts. The concept of soft set theory was introduced by Molodtsov [3] in 1999 and it became an important tool to manage mathematical uncertainties. Das and Samanta [4] in 2013 developed a new class of metric space named soft metric based on soft points of soft sets. Fixed point results of D-metric spaces and soft metric spaces can be seen in [5–8] and in other related papers of the same authors. The scope of soft D-metric was first investigated by Gunduz Aras et al. [8] and they proposed the notion of Δ-distance on a complete soft D-metric and gave a fixed point theorem on it.

In this paper we are presenting the concept of orbit for soft points, and then we are defining soft D_a-metric for the case of a single parameter a as can be seen in [6]. And for the single

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parameter case of soft D_a-metric space we establish several fixed point results and its general principles with valid proofs.

The main results of this paper are divided into two sections. In Section 3, we define orbit and related terminologies. Later we discuss an important general fixed point principle and some general results as corollaries. Section 4 deals with α_a-condensing maps and fixed point results involving α_a-condensing maps in soft D_a-metric space.

2. Priliminaries

Here are a few definitions that are necessary to understand the main points in the following sections.

Definition 2.1 ([3]) *Let X be a universal set, E be non-empty set of all parameters and $P(X)$ be the power set of X. A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \longrightarrow P(X)$. That is we can understand a soft set as a parametrized family of subsets of the set X. For each $a \in E$, $F(a)$ denotes the set of a-elements or a-approximate elements of the soft set (F, E).*

Definition 2.2 ([9]) *A soft set (F, E) over X is said to be an absolute soft set if for all $a \in E$, $F(a) = X$. It is denoted by $\tilde{X}$.*

Definition 2.3 *A soft set (F, E) over X is said to be a soft point if there is exactly one $a \in E$, such that $F(a) = x$ for some $x \in X$ and $F(b) = \phi$, $\forall\, b \in E \setminus \{x\}$.*

Definition 2.4 ([2]) *Let X be a non empty set. A function $D : X^3 \longrightarrow [0, \infty)$ is called a D-metric if the following conditions are satisfied:*

(1) $D(x, y, z) \geq 0$ for all $x, y, z \in X$ and equality holds if and only if $x = y = z$;

(2) $D(x, y, z) = D(x, z, y) = D(y, x, z) = \cdots$;

(3) $D(x, y, z) \leq D(x, y, u) + D(x, u, z) + D(u, y, z)$, for all $x, y, z, u \in X$.

Then the pair (X, D) is called a D-metric space.

So here onwards let us denote the absolute soft set by $\tilde{X}$, the collection of all soft points of $\tilde{X}$ by $SP(\tilde{X})$ and E be a non empty set of parameters.

Definition 2.5 ([8]) *Let X be an initial universal set and E be the non-empty set of parameters. Let $\tilde{X}$ be the absolute soft set, $F(a) = \tilde{X}$, $\forall a \in E$, where $(F, E) = \tilde{X}$. Let $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$ and $R(E^*)$ denotes the set of all non negative soft real numbers. A mapping $D : SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \longrightarrow R(E^*)$ is called a soft D-metric on the soft set $\tilde{X}$ that satisfies the following conditions, for each soft points $x_a, y_b, z_c, u_d \in SP(\tilde{X})$,*

(1) $D(x_a, y_b, z_c) \geq \tilde{0}$ and equality holds if and only if $x_a = y_b = z_c$ (coincidence);

(2) $D(x_a, y_b, z_c) = D(y_b, x_a, z_c) = D(x_a, z_c, y_b) = \cdots$ (Symmetry);

(3) $D(x_a, y_b, z_c) \leq D(x_a, y_b, u_d) + D(x_a, u_d, z_c) + D(u_d, y_b, z_c)$.

Then the set of $\tilde{X}$ with a soft D-metric is called a soft D-metric space and denoted by $(\tilde{X}, D, E)$.

Definition 2.6 ([8]) *Let $(\tilde{X}, D, E)$ be a soft D-metric space. A soft sequence $\{x^n_{a_n}\}$ converges to $x_b \in SP(\tilde{X})$ if for each $\tilde{\epsilon} > \tilde{0}$, there exists $n_0 \in N$ such that $\forall m, n > n_0, D(x^n_{a_n}, x^m_{a_m}, x_b) < \tilde{\epsilon}$.*

Definition 2.7 ([8]) *A soft sequence $\{x^n_{a_n}\}$ is called a Cauchy sequence if for each $\tilde{\epsilon} > \tilde{0}$, there exists $n_0 \in N$ such that $\forall\, m > n,\ p > n_0,\ D(x^n_{a_n}, x^m_{a_m}, x^p_{a_p}) < \tilde{\epsilon}$.*

Definition 2.8 ([8]) *A soft metric space is complete if every Cauchy sequence is convergent.*

Definition 2.9 ([10]) *Let $f : X \longrightarrow X$. The orbit of f at the point $x \in X$ is the set $O(x) = \{x, fx, f^2x, \ldots\}$.*

Definition 2.10 ([10]) *An orbit of x is said to be bounded if there exists a constant $K > 0$ such that $D(u, v, w) \leq K$ for all $u, v, w \in O(x)$. The constant K is called D-bound of $(O(x))$.*

Definition 2.11 ([10]) *The D-metric is said to be f-orbitally bounded if $O(x)$ is bounded for each $x \in X$.*

3. Main results

This section uses an iterative procedure to provide the definition of orbit in the context of soft sets. Additionally, we look into the range of applications of fixed point theory in the context of metric spaces defined in a specific soft set orbit. We are confirming the conclusions that have already been established in the context of D-metric spaces. But this time, we are talking about a concept dealing with soft points. Theorems for the new domain and new metric structure must therefore be verified. In this section, we also define convergence, strong convergence and very strong convergence in the context of soft D_a-metric space. And using the soft convergence conditions, we prove the fixed point results.

Definition 3.1 *Let $\tilde{X}$ be an absolute soft set and $(\tilde{X}, D_a, E)$ be a soft D_a-metric space. Let $\tilde{f}$ be a self map on $SP(\tilde{X})$. The orbit of $\tilde{f}$ at the point x_a is the set $O(\tilde{x}_a) = \{\tilde{x}_a, \tilde{f}\tilde{x}_a, \tilde{f}^2\tilde{x}_a, \ldots\}$.*

Definition 3.2 *An orbit of $\tilde{x}_a$ is said to be bounded if there exists a constant $K_a > \tilde{0}$ such that $D_a(\alpha_a, \beta_a, \gamma_a) \leq K_a$ for all $\alpha_a, \beta_a, \gamma_a \in O(\tilde{x}_a)$. The constant K_a is called a D_a-bound of $O(\tilde{x}_a)$.*

Definition 3.3 *A soft D_a-metric space $(\tilde{X}, D_a, E)$ is said to be $\tilde{f}$ orbitally bounded if $O(\tilde{x}_a)$ is bounded for each $\tilde{x}_a \in SP(\tilde{X})$.*

Definition 3.4 ([7]) *Let $(\tilde{X}, D_a, E)$ be a soft D_a-metric space.*

(i) A soft sequence $\{x^n_{a_n}\}$ in $(\tilde{X}, D_a, E)$ converges to a soft point $x_b \in SP(\tilde{X})$ if for each $\tilde{\epsilon} > \tilde{0}$, there exists $n_0 \in N$ such that $\forall\, m, n > n_0, D(x^n_{a_n}, x^m_{a_m}, x_b) < \tilde{\epsilon}$.

(ii) A soft sequence $\{x^n_{a_n}\}$ in $(\tilde{X}, D_a, E)$ is called a D_a-Cauchy sequence if for each $\tilde{\epsilon} > \tilde{0}$, there exists $n_0 \in N$ such that $\forall\, m > n, p > n_0,\ D(x^n_{a_n}, x^m_{a_m}, x^p_{a_p}) < \tilde{\epsilon}$.

(iii) A soft D_a-metric space $(\tilde{X}, D_a, E)$ is complete if every D_a-Cauchy sequence is convergent.

Definition 3.5 ([11]) *Let $(\tilde{X}, D_a, E)$ be a soft D_a-metric space and $\{x^n_{a_n}\}$ be a soft sequence in $(\tilde{X}, D_a, E)$, we say that $\{x^n_{a_n}\}$ converges strongly to an element x_b in $SP(\tilde{X})$ if*

(i) If for each $\tilde{\epsilon} > \tilde{0}$, there exists $n_0 \in N$ such that $\forall\, m > n, p > n_0$, $D(x^n_{a_n}, x^m_{a_m}, x_b) < \tilde{\epsilon}$.

(ii) $\{D_a(y_b, y_b, x^n_{a_n})\}$ converges to $D_a(y_b, y_b, x_b)$ for all $y_b \in SP(\tilde{X})$.

Definition 3.6 *Let $(\tilde{X}, D_a, E)$ be a soft D_a-metric space and $\{x^n_{a_n}\}$ be a soft sequence in $(\tilde{X}, D_a, E)$, we say that $\{x^n_{a_n}\}$ converges strongly to an element x_b in $SP(\tilde{X})$ if*

(i) If for each $\tilde{\epsilon} > \tilde{0}$, there exists $n_0 \in N$ such that $\forall\, m > n, p > n_0$, $D(x^n_{a_n}, x^m_{a_m}, x_b) < \tilde{\epsilon}$.

(ii) $\{D_a(y_b, z_b, x^n_{a_n})\}$ converges to $D_a(y_b, z_b, x_b)$ for all y_b, $z_b \in SP(\tilde{X})$.

Remark 3.7 It is clear that very strong convergence implies convergence and not the converse. It is also clear that in a soft D_a-metric space, every strongly convergent sequence has a unique strong limit where as limits are not unique under convergence.

Definition 3.8 *An orbit $O(\tilde{x}_a)$ is called $\tilde{f}$ orbitally complete if every D_a cauchy sequence in $O(\tilde{x}_a)$ strongly converges to a strong limit in $SP(\tilde{X})$.*

Lemma 3.9 *Let $\{\tilde{x}^n_{a_n}\}$ be a bounded soft sequence with D_a bound K_a satisfying*

$$D_a(\tilde{x}^n_{a_n}, \tilde{x}^{n+1}_{a_{n+1}}, \tilde{x}^m_{a_m}) \leq \lambda_a^n K_a$$

for all positive integers $m > n$ and some $\tilde{0} \leq \lambda_a < \tilde{1}$. Then $\{\tilde{x}^n_{a_n}\}$ is D_a-Cauchy.

Theorem 3.10 *Let $(\tilde{X}, D_a, E)$ be a soft D_a-metric space, $\tilde{f}$ be a self map of $SP(\tilde{X})$. Suppose that there exists an $\tilde{x}_a \in SP(\tilde{X})$ such that $O(\tilde{x}_a)$ is D_a bounded and $\tilde{f}$-orbitally complete. Suppose also that $\tilde{f}$ satisfies*

$$D_a(\tilde{f}\tilde{x}, \tilde{f}\tilde{y}, \tilde{f}\tilde{z}) \leq \lambda_a \max\{D_a(\tilde{x}, \tilde{y}, \tilde{z}), D_a(\tilde{x}, \tilde{f}\tilde{x}, \tilde{z})\} \tag{3.1}$$

for $\tilde{x}, \tilde{y}, \tilde{z} \in \overline{O(\tilde{x}_a)}$ for some $\tilde{0} \leq \lambda_a < \tilde{1}$. Then $\tilde{f}$ has a unique fixed point in $SP(\tilde{X})$.

Proof Suppose that there exists an n such that $\tilde{x}_{a_n} = \tilde{x}_{a_{n+1}}$. Then $\tilde{f}$ has $\tilde{x}_{a_n}$ as a fixed point in $SP(\tilde{X})$. Therefore, we may assume that all of the $\tilde{x}_{a_n}$ are distinct. We wish to show that, for any positive integers $m, n, m > n$, that

$$D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_m}) \leq \lambda_a^n K_a,$$

where K_a is the D_a bound of $O(x_a)$. The proof is by induction. From (3.1), for any m,

$$D_a(\tilde{x}_{a_1}, \tilde{x}_{a_2}, \tilde{x}_{a_m}) \leq \lambda_a \max\{D_a(\tilde{x}_{a_0}, \tilde{x}_{a_1}, \tilde{x}_{a_{m-1}}), D_a(\tilde{x}_{a_0}, \tilde{x}_{a_1}, \tilde{x}_{a_{m-1}})\} \leq \lambda_a K_a. \tag{3.2}$$

Again using (3.1)

$$D_a(\tilde{x}_a, \tilde{x}_{a_3}, \tilde{x}_{a_m}) \leq \lambda_a \max\{D_a(\tilde{x}_{a_2}, \tilde{x}_{a_3}, \tilde{x}_{a_{m-1}}), D_a(\tilde{x}_{a_1}, \tilde{x}_{a_2}, \tilde{x}_{a_{m-1}})\}. \tag{3.3}$$

Using (3.2)

$$D_a(\tilde{x}_{a_2}, \tilde{x}_{a_3}, \tilde{x}_{a_m}) \leq \lambda_a \max\{D_a(\tilde{x}_{a_2}, \tilde{x}_{a_3}, \tilde{x}_{a_{m-1}}), \lambda_a K_a\}. \tag{3.4}$$

Inequality (3.4) can be regarded as a recursion formula in m. Therefore,

$$D_a(\tilde{x}_{a_2}, \tilde{x}_{a_3}, \tilde{x}_{a_m}) \leq \lambda_a \max\{\lambda_a \max\{D_a(\tilde{x}_{a_2}, \tilde{x}_{a_3}, \tilde{x}_{a_{m-2}}), \lambda_a K_a\}, \lambda_a K_a\} \leq \lambda_a^2 K_a. \tag{3.5}$$

Assume the induction hypothesis. Then from (3.1),

$$\begin{aligned}
&D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_m})\\
&\quad\leq \lambda_a \max\{D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-1}}), D_a(\tilde{x}_{a_n}, \tilde{x}_{a_{n+1}}, \tilde{x}_{a_{m-1}})\}\\
&\quad\leq \lambda_a \max\{D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-1}}), \lambda_a^n K_a\}. \qquad (3.6)
\end{aligned}$$

Inequality (3.6) can be regarded as a recursion formula in m. Therefore,

$$\begin{aligned}
&D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_m})\\
&\quad\leq \lambda_a \max\{\lambda_a \max\{D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-2}}), \lambda_a^n K_a\}, \lambda_a^n K_a\}\\
&\quad= \max\{\lambda_a^2 D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-2}}), \lambda_a^{n+2} K_a, \lambda_a^{n+1} K_a\}\\
&\quad= \max\{\lambda_a^2 D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-2}}), \lambda_a^{n+1} K_a\}\\
&\quad\leq \max\{\lambda_a^2 \lambda_a \max\{D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-3}}), \lambda_a^{n+1} K_a\}, \lambda_a^{n+1} K_a\}\\
&\quad= \max\{\lambda_a^3 D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-3}}), \lambda_a^{n+1} K_a\} \leq \cdots\\
&\quad\leq \max\{\lambda_a^n D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-n}}), \lambda_a^{n+1} K_a\}\\
&\quad\leq \max\{\lambda_a^n \lambda_a \max\{D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+2}}, \tilde{x}_{a_{m-n-1}}), \lambda_a^{n+1} K_a\}, \lambda_a^{n+1} K_a\}\\
&\quad= \lambda_a^{n+1} K_a \qquad (3.7)
\end{aligned}$$

and $\{\tilde{x}^n_{a_n}\}$ is D_a-Cauchy by Lemma 3.9. Since $SP(\tilde{X})$ is x_a-orbitally complete, there exists a $\tilde{p_a} \in SP(\tilde{X})$ with $\lim \tilde{x}^n_{a_n} = \tilde{p_a}$. In (3.1) set $\tilde{x} = \tilde{x}_{a_n}$, $\tilde{z} = \tilde{p_a}$ to obtain

$$D_a(\tilde{x}_{a_{n+1}}, \tilde{x}_{a_{n+1}} \tilde{f}\tilde{p_a}) \leq \lambda_a \max\{D_a(\tilde{x}_{a_n}, \tilde{x}_{a_{n+1}}, \tilde{p_a}), D_a(\tilde{x}_{a_n}, \tilde{x}_{a_{n+1}}, \tilde{p_a})\}. \qquad (3.8)$$

Taking the limit of (3.8) as $n \longrightarrow \infty$ yields

$$D_a(\tilde{p_a}, \tilde{p_a}, \tilde{f}\tilde{p_a}) \leq \lambda_a D_a(\tilde{p_a}, \tilde{p_a}, \tilde{p_a}) = \tilde{0}$$

and $\tilde{p_a} = \tilde{f}\tilde{p_a}$. To prove uniqueness, suppose that $\tilde{q_a}$ is also a fixed point of $\tilde{f}$. Then from (3.1)

$$\begin{aligned}
&D_a(\tilde{p_a}, \tilde{p_a}, \tilde{q_a}) = D_a(\tilde{f}\tilde{p_a}, \tilde{f}\tilde{p_a}, \tilde{f}\tilde{q_a})\\
&\quad\leq \lambda_a \max\{D_a(\tilde{p_a}, \tilde{p_a}, \tilde{q_a}), D_a(\tilde{p_a}, \tilde{f}\tilde{p_a}, \tilde{q_a})\} = \lambda_a D_a(\tilde{p_a}, \tilde{p_a}, \tilde{q_a}), \qquad (3.9)
\end{aligned}$$

which implies that $\tilde{p_a} = \tilde{q_a}$. □

Corollary 3.11 *Let $\tilde{f}$ be a selfmap of a complete and bounded soft D_a-metric space $SP(\tilde{X})$ satisfying*

$$D_a(\tilde{f}\tilde{x}_a, \tilde{f}\tilde{y}_a, \tilde{f}\tilde{z}_a) \leq \lambda_a D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a), \qquad (3.10)$$

for all $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in SP(\tilde{X})$, for some $\tilde{0} \leq \lambda_a < \tilde{1}$. Then $\tilde{f}$ has a unique fixed point $\tilde{p_a}$, and $\tilde{f}$ is continuous at $\tilde{p_a}$.

Proof In (3.10) set $\tilde{y_a} = \tilde{f}\tilde{x_a}$ to obtain (3.1). Then, from Theorem 3.10, $\tilde{f}$ has a unique fixed point $\tilde{p_a}$. To prove continuity, let $\{\tilde{z}^n_{a_n}\} \subset SP(\tilde{X})$ with $\lim \tilde{z}^n_{a_n} = \tilde{p_a}$. From (3.10),

$$D_a(\tilde{p_a}, \tilde{p_a}, \tilde{f}\tilde{z}_{a_n}) = D_a(\tilde{f}\tilde{p_a}, \tilde{f}p_a, \tilde{f}\tilde{z}_{a_n}) \leq \lambda_a D_a(\tilde{p_a}, \tilde{p_a}, \tilde{f}\tilde{z}_{a_n}). \qquad (3.11)$$

Taking the limit as $n \longrightarrow \infty$ gives $\limsup D_a(\tilde{p_a}, \tilde{p_a}, \tilde{f}\tilde{z}_{a_n}) = \tilde{0}$ and $\liminf D_a(\tilde{p_a}, \tilde{p_a}, \tilde{f}\tilde{z}_{a_n}) = \tilde{0}$ which implies that $\lim \tilde{f}\tilde{z}_{a_n} = \tilde{p_a} = \tilde{f}\tilde{p_a}$ and $\tilde{f}$ is continuous at $\tilde{p_a}$.

Corollary 3.12 *Let $\tilde{f}$ be a selfmap of a complete and bounded soft D_a-metric space satisfying the condition that there exists a positive integer m*

$$D_a(\tilde{f}^m\tilde{x_a}, \tilde{f}^m\tilde{y_a}, \tilde{f}^m\tilde{z_a}) \le \lambda_a D_a(\tilde{x_a}, \tilde{y_a}, \tilde{z_a}), \tag{3.12}$$

for all $\tilde{x_a}, \tilde{y_a}, \tilde{z_a} \in SP(\tilde{X})$, for some $\tilde{0} \le \lambda < \tilde{1}$. Then $\tilde{f}$ has a unique fixed point $\tilde{p_a}$ and $\tilde{f}$ is orbitally continuous at $\tilde{p_a}$.

Proof Define $\tilde{T} = \tilde{f}^m$. Then (3.12) reduces to (3.10), and $\tilde{T}$ has a unique fixed point $\tilde{p_a}$ by previous corollary; i.e., $\tilde{p_a} = \tilde{T}\tilde{p_a} = \tilde{f}^m\tilde{p_a}$. Thus $\tilde{f}\tilde{p_a} = \tilde{f}^{m+1}\tilde{p_a} = \tilde{T}(\tilde{f}\tilde{p_a})$ and $\tilde{f}\tilde{p_a}$ is also a fixed point of $\tilde{T}$. Uniqueness implies that $\tilde{f}\tilde{p_a} = \tilde{p_a}$ and $\tilde{p_a}$ is a fixed point of $\tilde{f}$. For the continuity, let $\{\tilde{z}_{a_n}\} \subset O(\tilde{f})$, with $\lim \tilde{z}_{a_n} = \tilde{p_a}$. From (3.12),

$$D_a(\tilde{f}^m\tilde{p_a}, \tilde{f}^m\tilde{p_a}, \tilde{f}^m\tilde{z}_{a_n}) \le \lambda_a D_a(\tilde{p_a}, \tilde{p_a}, \tilde{z}_{a_n}). \tag{3.13}$$

Taking the limit as $n \longrightarrow \infty$ shows that $\lim \tilde{f}^m\tilde{z}_{a_n} = \tilde{p_a} = \tilde{f}^m\tilde{p_a}$ and $\tilde{f}^m$ is $\tilde{f}$ orbitally continuous at $\tilde{p_a}$. But since each $\tilde{z}_{a_n} \in O(\tilde{f}), \lim \tilde{f}^m\tilde{z}_{a_n} = \tilde{f}\tilde{z}_{a_{n+m-1}}$ and $\tilde{f}$ is $\tilde{f}$-orbitally continuous at $\tilde{p_a}$. □

Corollary 3.13 *Let $\tilde{f}$ be a self map of $SP(\tilde{X})$, $SP(\tilde{X})$ an $\tilde{f}$-orbitally bounded and soft D_a-metric space satisfying*

$$D_a(\tilde{f}\tilde{x_a}, \tilde{f}\tilde{y_a}, \tilde{f}\tilde{z_a}) \le \alpha_a[\frac{\tilde{1} + D_a(\tilde{x_a}, \tilde{f}\tilde{x_a}, \tilde{z_a})}{\tilde{1} + D_a(\tilde{x_a}, \tilde{y_a}, \tilde{z_a})}]D_a(\tilde{y_a}, \tilde{f}\tilde{y_a}, \tilde{z_a}) + \beta_a D_a(\tilde{x_a}, \tilde{y_a}, \tilde{z_a}), \tag{3.14}$$

for all $\tilde{x_a}, \tilde{y_a}, \tilde{z_a} \in SP(\tilde{X})$, $\alpha_a + \beta_a < \tilde{1}$. Then $\tilde{f}$ has a unique fixed point $\tilde{p_a}$ and $\tilde{f}$ is continuous at $\tilde{p_a}$.

Proof In (3.14) set $\tilde{y} = \tilde{f}\tilde{x}$ to obtain

$$\begin{aligned} D_a(\tilde{f}\tilde{x_a}, \tilde{f}^2\tilde{x_a}, \tilde{f}\tilde{z_a}) &\le \alpha_a D_a(\tilde{f}\tilde{x_a}, \tilde{f}^2\tilde{x_a}, \tilde{z_a}) + \beta_a D_a(\tilde{x_a}, \tilde{f}\tilde{x_a}, \tilde{z_a}) \\ &\le \lambda_a \max\{D_a(\tilde{f}\tilde{x_a}, \tilde{f}^2\tilde{x_a}, \tilde{z_a}), D_a(\tilde{x_a}, \tilde{f}\tilde{x_a}, \tilde{z_a})\}, \end{aligned} \tag{3.15}$$

where $\lambda_a = \alpha_a + \beta_a < \tilde{1}$, and (3.1) is satisfied. The conclusion follows from Theorem 3.10. To prove the continuity of $\tilde{f}$ at $\tilde{p_a}$, let $\{\tilde{z}^n_{a_n}\} \subset SP(\tilde{X})$ with $\lim \tilde{z}_{a_n} = \tilde{p_a}$. In (3.14) set $\tilde{x_a} = \tilde{z_a} = \tilde{p_a}$, $\tilde{y_a} = \tilde{z}_{a_n}$, to obtain

$$\begin{aligned} D_a(\tilde{p_a}, \tilde{f}\tilde{z}_{a_n}, \tilde{p_a}) &\le D_a(\tilde{f}\tilde{p_a}, \tilde{f}\tilde{z}_{a_n}, \tilde{f}\tilde{p_a}) \\ &\le \alpha_a[\frac{\tilde{1} + D_a(\tilde{p_a}, \tilde{f}\tilde{p_a}, p_a)}{\tilde{1} + D_a(\tilde{p_a}, \tilde{f}\tilde{z}_{a_n}, \tilde{p_a})}]D_a(\tilde{z}_{a_n}, \tilde{f}\tilde{z}_{a_n}, \tilde{p_a}) + \beta_a D_a(\tilde{p_a}, \tilde{f}\tilde{z}_{a_n}, \tilde{p_a}) \\ &\le \alpha_a D_a(\tilde{z}_{a_n}, \tilde{f}\tilde{z}_{a_n}, \tilde{p_a}) + \beta_a D_a(\tilde{p_a}, \tilde{z}_{a_n}, \tilde{p_a}). \end{aligned} \tag{3.16}$$

Taking the lim sup of both sides of (3.16) as $n \longrightarrow \infty$ yields

$$D_a(\tilde{p_a}, \limsup \tilde{f}\tilde{z}_{a_n}, \tilde{p_a}) \le \alpha_a D_a(\tilde{p_a}, \limsup \tilde{f}\tilde{z}_{a_n}, \tilde{p_a}), \tag{3.17}$$

which implies that $\limsup \tilde{f}\tilde{z}_{a_n} = \tilde{p_a}$. Similarly, taking the $\liminf$ of both sides of (3.16) as $n \longrightarrow \infty$ yields

$$D_a(\tilde{p_a}, \liminf \tilde{f}\tilde{z}_{a_n}, \tilde{p_a}) \leq \alpha_a D_a(p_a, \liminf \tilde{f}\tilde{z}_{a_n}, \tilde{p_a}), \tag{3.18}$$

which implies that $\liminf \tilde{f}\tilde{z}_{a_n} = \tilde{p_a}$. Therefore, $\lim \tilde{f}\tilde{z}_{a_n} = \tilde{p_a} = \tilde{f}\tilde{p_a}$ and $\tilde{f}$ is continuous at $\tilde{p_a}$.

Corollary 3.14 *Let $\tilde{f}$ be a self map of an $\tilde{f}$ orbitally bounded and complete soft D_a-metric space $SP(\tilde{X})$, m a fixed positive integer. Suppose that $\tilde{f}$ satisfies*

$$\begin{aligned} D_a(\tilde{f}^m\tilde{x}_a, \tilde{f}^m\tilde{y}_a, \tilde{f}^m\tilde{z}_a) \leq &\alpha_a[\frac{\tilde{1} + D_a(\tilde{x}_a, \tilde{f}^m\tilde{x}_a, \tilde{z}_a)}{\tilde{1} + D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a)}]D_a(\tilde{y}_a, \tilde{f}^m\tilde{y}_a, \tilde{z}_a)+ \\ &\beta_a D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a), \end{aligned} \tag{3.19}$$

for all $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in SP(\tilde{X})$, $\alpha_a + \beta_a < \tilde{1}$. Then $\tilde{f}$ has a unique fixed point $\tilde{p_a}$ and $\tilde{f}$ is $\tilde{f}$ orbitally continuous at $\tilde{p_a}$.

Proof Set $\tilde{T} = \tilde{f}^m$. Then $\tilde{T}$ satisfies (3.14). Therefore, $\tilde{T}$ has a unique fixed point at $\tilde{p_a}$ and is continuous at $\tilde{p_a}$. □

4. α_a-condensing

Now we can define α_a-condensing for the case of an orbit $O(\tilde{x}_a)$ in a soft D_a-metric space $SP(\tilde{X})$ and follow related results.

Definition 4.1 *For any set $O(\tilde{x}_a)$ in a soft D_a-metric space $SP(\tilde{X})$, the D_a-diameter of $O(\tilde{x}_a)$, denoted by $\delta_a O(\tilde{x}_a)$, is defined by $\delta_a O(\tilde{x}_a) = \sup_{\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in O(\tilde{x}_a)} D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a)$. The measure of non compactness of an orbit $O(\tilde{x}_a)$ in a soft D_a-metric space $SP(\tilde{X})$ is a non negative real number $\alpha_a(O(\tilde{x}_a))$ defined by*

$$\alpha_a(O(\tilde{x}_a)) = \inf \left\{ \begin{array}{ll} \gamma_a > \tilde{0} : O(\tilde{x}_a) = \bigcup_{i=1}^n O(\tilde{x}_{ai}) & \text{for which } \delta_a(O(\tilde{x}_{ai})) \leq \gamma_a \\ & \text{for } i = 1, 2, \ldots, n \end{array} \right\}. \tag{4.1}$$

Definition 4.2 *A self map $\tilde{f}$ of $SP(\tilde{X})$ is called α_a-condensing, if for any bounded orbit $O(\tilde{x}_a)$ in $SP(\tilde{X})$ is bounded*

$$\alpha_a(\tilde{f}(O(\tilde{x}_a))) < \alpha_a(O(\tilde{x}_a))$$

if $\alpha_a(O(\tilde{x}_a)) > \tilde{0}$.

Lemma 4.3 *Let $\tilde{f} : SP(\tilde{X}) \longrightarrow SP(\tilde{X})$, $SP(\tilde{X})$ an $\tilde{f}$-orbitally bounded and complete soft D_a-metric space be α_a-condensing. Then $\overline{O(\tilde{x}_a)}$ is compact for each $\tilde{x}_a \in SP(\tilde{X})$.*

Theorem 4.4 *Let $\tilde{f}$ be a continuous compact selfmap of a bounded soft D_a-metric space $SP(\tilde{X})$ satisfying*

$$D_a(\tilde{f}^r\tilde{x}_a, \tilde{f}^s\tilde{y}_a, \tilde{f}^t\tilde{z}_a) < \delta_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a) \text{ for each } \tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in SP(\tilde{X}) \tag{4.2}$$

with two of $\{\tilde{x}_a, \tilde{y}_a, \tilde{z}_a\}$ distinct, where r, s, t are fixed positive integers. Then $\tilde{f}$ has a unique fixed point in $SP(\tilde{X})$.

Proof Since $\tilde{f}$ is compact, there exists a compact subset $SP(\tilde{Y})$ of $SP(\tilde{X})$ containing $\tilde{f}(SP(\tilde{X}))$. Then $\tilde{f}(SP(\tilde{Y})) \subset \tilde{Y}$ and $\tilde{A} := \cap_{n=1}^{\infty}\tilde{f}^n SP(\tilde{Y})$ is a non empty compact $\tilde{f}$-invariant subset of $SP(\tilde{X})$ which is mapped by $\tilde{f}$ onto itself. $\tilde{A}$ has the same properties with respect to $\tilde{f}^r, \tilde{f}^s$, and $\tilde{f}^t$. Suppose that $\delta(\tilde{A}) > \tilde{0}$. Since $\tilde{A}$ is compact there exist $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in \tilde{A}$ such that $\delta(\tilde{A}) = D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a)$. Since $\tilde{f}\tilde{A} = \tilde{A}$, there exist $\tilde{x}'_a, \tilde{y}'_a$ and $\tilde{z}'_a$ in $\tilde{A}$ such that $\tilde{x}_a = \tilde{f}^r\tilde{x}'_a, \tilde{y}_a = \tilde{f}^s\tilde{y}'_a$ and $\tilde{z}_a = \tilde{f}^t\tilde{z}'_a$. Then from (4.2),

$$\delta(\tilde{A}) = D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a) = D_a(\tilde{f}^r\tilde{x}'_a, \tilde{f}^s\tilde{y}'_a, \tilde{f}^t\tilde{z}'_a) < \delta(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a) = \delta(\tilde{A}), \tag{4.3}$$

a contradiction. Therefore, $\tilde{A}$ consists of a single point, which is a fixed point of $\tilde{f}$.

Suppose $\tilde{p}_a$ and $\tilde{q}_a$ are fixed points of $\tilde{f}, \tilde{p}_a \neq \tilde{q}_a$. Then from (4.2)

$$\tilde{0} < D_a(\tilde{p}_a, \tilde{p}_a, \tilde{q}_a) = D_a(\tilde{f}^r\tilde{p}_a, \tilde{f}^s\tilde{p}_a, \tilde{f}^t\tilde{q}_a) < D_a(\tilde{p}_a, \tilde{p}_a, \tilde{q}_a), \tag{4.4}$$

a contradiction. Therefore, the fixed point is unique. □

Corollary 4.5 *Let $SP(\tilde{X})$ be a compact D_a-metric space, $\tilde{f}$, a continuous selfmap of $SP(\tilde{X})$ satisfying*

$$D_a(\tilde{f}\tilde{x}_a, \tilde{f}\tilde{y}_a, \tilde{f}\tilde{z}_a) < \max\left\{\begin{array}{l} D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a), D_a(\tilde{x}_a, \tilde{f}\tilde{x}_a, \tilde{z}_a), \\ D_a(\tilde{y}_a, \tilde{f}\tilde{y}_a, \tilde{z}_a), D_a(\tilde{x}_a, \tilde{f}\tilde{y}_a, \tilde{z}_a), \\ D_a(\tilde{y}_a, \tilde{f}\tilde{x}_a, \tilde{z}_a) \end{array}\right\} D_a(\tilde{p}_a, \tilde{p}_a, \tilde{q}_a), \tag{4.5}$$

for all $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in SP(\tilde{X})$ with $\tilde{x}_a \neq \tilde{f}\tilde{x}_a, \tilde{y}_a \neq \tilde{f}\tilde{y}_a, \tilde{z}_a \neq \tilde{f}\tilde{z}_a$. Then $\tilde{f}$ has a unique fixed $\tilde{p}_a$ in $SP(\tilde{X})$.

Proof Inequality (4.5) implies that $D_a(\tilde{f}\tilde{x}_a, \tilde{f}\tilde{y}_a, \tilde{f}\tilde{z}_a) < \delta_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a)$ and the existense and uniqueness of a fixed point $\tilde{p}_a$ follows from Theorem 4.4.

For continuity, let $\{\tilde{z}_{a_n}\} \subset SP(\tilde{X})$ with $\tilde{z}_{a_n} \neq \tilde{p}_a$ for each n and $\lim \tilde{z}_{a_n} = \tilde{p}_a$. From (4.5)

$$D_a(\tilde{p}_a, \tilde{p}_a, \tilde{f}\tilde{z}_{a_n}) = D_a(\tilde{f}\tilde{p}_a, \tilde{f}\tilde{p}_a, \tilde{f}\tilde{z}_{a_n}) < D_a(\tilde{p}_a, \tilde{f}\tilde{p}_a, \tilde{z}_{a_n}) \tag{4.6}$$

taking the limit as $n \longrightarrow \infty$ implies that $\tilde{f}$ is continuous at p_a. □

Theorem 4.6 *Let $\tilde{f}$ be an $\tilde{f}$-orbitally continuous α_a-condensing selfmap of a complete bounded soft D_a-metric space $SP(\tilde{X})$. Let $\tilde{x}_a \in SP(\tilde{X})$. If (4.2) holds on $\overline{O(\tilde{x}_a)}$, then $\tilde{f}$ has a unique fixed point $\tilde{p}_a \in \overline{O(\tilde{x}_a)}$ and $\lim_n \tilde{f}^n\tilde{x}_a = \tilde{p}_a$ for each $\tilde{x}_a \in \overline{O(\tilde{x}_a)}$.*

Proof From (4.3), $\overline{O(\tilde{x}_a)}$ is compact. Since $\tilde{f}$ is a continuous α_a-condensing selfmap of $\overline{O(\tilde{x}_a)}$, $\tilde{f}$ is compact. Now apply Theorem 4.4. □

Corollary 4.7 *Let $\tilde{f}$ be a continuous α_a-condensing selfmap of a complete bounded soft D_a-metric space $SP(\tilde{X})$ satisfying (4.5) for all $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in SP(\tilde{X})$ with $\tilde{x}_a \neq \tilde{f}\tilde{x}_a, \tilde{y}_a \neq \tilde{f}\tilde{y}_a, \tilde{z}_a \neq \tilde{f}\tilde{z}_a$. Then $\tilde{f}$ has a unique fixed point $\tilde{p}_a$ in $SP(\tilde{X})$.*

From Corollary 4.5, $D_a(\tilde{f}\tilde{x}_a, \tilde{f}\tilde{y}_a, \tilde{f}\tilde{z}_a) < \delta_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a)$ and Theorem 4.6 proves the Corollary.

Theorem 4.8 *Let $\tilde{f}$ be a selfmap of a soft D_a-metric space $SP(\tilde{X})$. Suppose that there exists a point $\tilde{x}_a \in SP(\tilde{X})$ with $\overline{O(\tilde{x}_a)}$ bounded and complete. Suppose that $\tilde{f}$ is continuous and*

α_a-condensing on $\overline{O(\tilde{x}_a)}$ and satisfies (4.2) for each $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in \overline{O(\tilde{x}_a)}$ with two of $\{\tilde{x}_a, \tilde{y}_a, \tilde{z}_a\}$ distinct, and $\tilde{x}_a \neq \tilde{f}\tilde{x}_a, \tilde{y}_a \neq \tilde{f}\tilde{y}_a, \tilde{z}_a \neq \tilde{f}\tilde{z}_a$. Then $\tilde{f}$ has a fixed point in $\overline{O(\tilde{x}_a)}$.

Proof By Lemma 4.3, $\overline{O(\tilde{x}_a)}$ is compact. If there exists some integer n for which $\tilde{f}^n\tilde{x}_a = \tilde{f}^{n+1}\tilde{x}_a$, then $\tilde{f}$ has a fixed point in $\overline{O(\tilde{x}_a)}$. Assume that $\tilde{f}^n\tilde{x}_a \neq \tilde{f}^{n+1}\tilde{x}_a$ for each n. Note that $\tilde{f}$ restricted to $\overline{O(\tilde{x}_a)}$ is a continuous compact selfmap of $\overline{O(\tilde{x}_a)}$. Suppose $\tilde{u}_a \neq \tilde{f}\tilde{u}_a$ for each cluster point u_a of $\overline{O(\tilde{x}_a)}$. Then $\tilde{f}$ satisfies condition (4.2) for all $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in \overline{O(\tilde{x}_a)}$, with two of $\{\tilde{x}_a, \tilde{y}_a, \tilde{z}_a\}$ distinct. Therefore, by Theorem 4.6, $\tilde{f}$ restricted to $\overline{O(\tilde{x}_a)}$ has a unique fixed point $p_a \in \overline{O(\tilde{x}_a)}$. This contradicts the assumption that $\tilde{u}_a \neq \tilde{f}\tilde{u}_a$ for each cluster point $\tilde{u}_a$ of $\overline{O(\tilde{x}_a)}$. Therefore, $\tilde{u}_a = \tilde{f}\tilde{u}_a$ for some cluster point $\tilde{u}_a \in \overline{O(\tilde{x}_a)}$. $\square$

The proofs of Theorems 4.4, 4.6 and 4.8 are very similar to their metric space counterparts in [3], and [12], but given here for completeness.

Theorem 4.9 *Let $\tilde{f}$ be a selfmap of $SP(\tilde{X})$, and $\tilde{f}$-orbitally bounded and complete soft D_a-metric space. Suppose that $\tilde{f}$ is α_a-condensing, $\tilde{f}$-orbitally bounded and continuous and satisfies*

$$\begin{aligned} D_a(\tilde{f}\tilde{x}_a, \tilde{f}\tilde{y}_a, \tilde{f}\tilde{z}_a) &\leq \alpha_a[\frac{\tilde{1} + D_a(\tilde{x}_a, \tilde{f}\tilde{x}_a, \tilde{z}_a)}{\tilde{1} + D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a)}]D_a(\tilde{y}_a, \tilde{f}\tilde{y}_a, \tilde{z}_a) + \beta_a D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a) \\ &= \tilde{M}(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a), \end{aligned} \tag{4.7}$$

for all $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in SP(\tilde{X})$ with $\tilde{x}_a \neq \tilde{f}\tilde{x}_a, \tilde{y}_a \neq \tilde{f}\tilde{y}_a, \tilde{z}_a \neq \tilde{f}\tilde{z}_a$, where $\alpha_a, \beta_a > \tilde{0}, \alpha_a + \beta_a \leq \tilde{1}$. Then $\tilde{f}$ has a unique fixed point $p_a \in SP(\tilde{X})$ and $\tilde{f}$ is continuous at p_a.

Proof If $\alpha_a + \beta_a < \tilde{1}$, the result follows from Corollary 3.14. Therefore, we assume that $\alpha_a + \beta_a = \tilde{1}$. Let $\tilde{x}_a \in SP(\tilde{X})$ and define $\tilde{x}_{a_{n+1}} = \tilde{f}\tilde{x}_{a_n}, n \geq 0$. From Lemma 4.3 it follows that $\overline{O(\tilde{x}_a)}$ is compact. Obviously, $\tilde{f} : \overline{O(\tilde{x}_a)} \longrightarrow \overline{O(\tilde{x}_a)}$.

Case I. There exist some $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in \overline{O(\tilde{x}_a)}$ for which $\tilde{M} = \tilde{0}$. Then $\tilde{y}_a = \tilde{f}\tilde{y}_a = \tilde{z}_a = \tilde{x}_a$, and $\tilde{y}_a$ is a fixed point of $\tilde{f}$. Inequality (4.7) implies uniqueness.

Case II. $\tilde{M} \neq \tilde{0}$ for all $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in \overline{O(\tilde{x}_a)}$. Define a function $\tilde{F} : (\overline{O(\tilde{x}_a)})^3 \longrightarrow [0, \infty)$ by

$$\tilde{F}(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a) = \frac{D_a(\tilde{f}\tilde{x}_a, \tilde{f}\tilde{y}_a, \tilde{f}\tilde{z}_a)}{M(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a)}. \tag{4.8}$$

The function $\tilde{F}$ is well defined on $(\overline{O(\tilde{x}_a)})^3$, since $\tilde{M} \neq \tilde{0}$ on $\overline{O(\tilde{x}_a)}$. Since $\tilde{F}$ is continuous on $\overline{O(\tilde{x}_a)}$, it attains its maximum value at some point $(\tilde{u}_a, \tilde{u}_a, \tilde{u}_a) \in \overline{O(\tilde{x}_a)}$. We call this maximum value $\tilde{c}$. From (4.7) it follows that $\tilde{0} < \tilde{c} < \tilde{1}$. Therefore,

$$\begin{aligned} D_a(\tilde{f}\tilde{x}_a, \tilde{f}\tilde{y}_a, \tilde{f}\tilde{z}_a) &\leq \tilde{c}M(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a) \\ &\leq \alpha'_a[\frac{\tilde{1} + D_a(\tilde{x}_a, \tilde{f}\tilde{x}_a, \tilde{z}_a)}{\tilde{1} + D_a(\tilde{x}_a, \tilde{x}_a, \tilde{z}_a)}]D_a(\tilde{y}_a, \tilde{f}\tilde{y}_a, \tilde{z}_a) + \beta'_a D_a(\tilde{x}_a, \tilde{y}_a, \tilde{z}_a), \end{aligned} \tag{4.9}$$

for all $\tilde{x}_a, \tilde{y}_a, \tilde{z}_a \in \overline{O(\tilde{x}_a)}$, where $\alpha'_a = \tilde{c}\alpha_a > \tilde{0}, \beta'_a = \tilde{c}\beta_a > \tilde{0}$ and $\alpha'_a + \beta'_a = \tilde{c}(\alpha_a + \beta_a) < \tilde{1}$. Since $\overline{O(\tilde{x}_a)}$ is compact, it is bounded and complete. The result follows from Corollary 3.13. $\square$

Corollary 4.10 *Let $\tilde{f}$ be a selfmap of a complete and $\tilde{f}$-orbitally bounded D_a-metric space. Suppose that $\tilde{f}$ is α_a-condensing and $\tilde{f}$-orbitally continuous. Let m be a positive integer. Suppose* *that $\tilde{f}$ satisfies*

$$D_a(\tilde{f}^m\tilde{x}_a,\tilde{f}^m\tilde{y}_a,\tilde{f}^m\tilde{z}_a)\leq \alpha_a\frac{\tilde{1}+D_a(\tilde{x}_a,\tilde{f}^m\tilde{x}_a,\tilde{z}_a)}{\tilde{1}+D_a(\tilde{x}_a,\tilde{y}_a,\tilde{z}_a)}]D_a(\tilde{y}_a,\tilde{f}^m\tilde{y}_a,\tilde{z}_a)+\beta_a D_a(\tilde{x}_a,\tilde{y}_a,\tilde{z}_a), \quad (4.10)$$

for all $\tilde{x}_a,\tilde{y}_a,\tilde{z}_a\in SP(\tilde{X})$ for which the right hand side of (4.10) is not zero, where α_a, $\beta_a>\tilde{0}$, $\alpha_a+\beta_a\leq\tilde{1}$. Then $\tilde{f}$ has a unique fixed point $\tilde{p}_a$ and $\tilde{f}$ is $\tilde{f}$ orbitally continuous at $\tilde{p}_a$.

Proof Set $\tilde{T}=\tilde{f}^m$. Then $\tilde{T}$ satisfies (4.7), and the existence and uniqueness of the fixed point $\tilde{p}_a$, for $\tilde{T}$, follows from (4.7). It then follows that $\tilde{p_a}$ is the unique fixed point for $\tilde{f}$. The continuity argument is the same as that used in the proof of Corollary 3.14. □

Corollary 4.11 *Let $\tilde{f}$ be a continuous selfmap of a compact D_a-metric space satisfying (4.7). Then $\tilde{f}$ has a unique fixed point $\tilde{p_a}$ and $\tilde{f}$ is continuous at $\tilde{p_a}$.*

This results is an immediate consequence of Theorem 4.8.

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