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Relative Rota-Baxter Operators on Hom-Lie-Yamaguti Algebras

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Abstract In this paper, we first introduce the notion of relative Rota-Baxter operators on Hom-Lie-Yamaguti algebras and give some characteristics of relative Rota-Baxter operators in terms of Nijenhuis operators and graphs. Then, the cohomology theory of relative Rota-Baxter operators on Hom-Lie-Yamaguti algebras is proposed. Finally, the deformation of the relative Rota-Baxter operator is explored by applying the cohomological approach.

Keywords Hom-Lie-Yamaguti algebra; relative Rota-Baxter operator; cohomology; deformation

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1. Introduction

Baxter [1] introduced the notion of Rota-Baxter operators on associative algebras when studying fluctuation problems in probability. Then Kupershmidt [2] introduced the notion of relative Rota-Baxter operator (also called $\mathcal{O}$-operator) on Lie algebra, which is a generalization of the Rota-Baxter operator, by finding that the relative Rota-Baxter operator is a solution in the classical Yang-Baxter equation. Further research on relative Rota-Baxter operators could be found in [3–8] and references cited therein.

In recent years, Hom-type algebras have been studied by many scholars. The first examples coming from q-deformations of Witt and Virasoro algebras are Hom-Lie algebras in [9]. The concept of Hom-Lie-Yamaguti algebra was introduced in [10]. It is a Hom-type generalization of a Lie-Yamaguti algebra of [11–15]. Recently, in [16], Ma and Chen studied one-parameter formal deformations of a Hom-Lie-Yamaguti algebra. In [17], Zhang and Li studied representations and cohomologies of a Hom-Lie-Yamaguti algebra. In [18], Dong and Ma studied linear deformations of a Hom-Lie-Yamaguti algebra and introduced the notions of a Nijenhuis operator, a product structure and a complex structure on a Hom-Lie-Yamaguti algebra. This paper aims to study the

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cohomology theory and deformations of relative Rota-Baxter operators on Hom-Lie-Yamaguti algebras based on some work in [3, 4, 16–18].

This paper is organized as follows. In Section 2, we recall some basic definitions of Hom-Lie-Yamaguti algebra. Section 3 introduces the notions of s-Rota-Baxter operators and Relative Rota-Baxter operators on Hom-Lie-Yamaguti algebras. Moreover, we give some characterisation of relative Rota-Baxter operators in terms of Nijenhuis operators and graphs. Section 4 introduces the cohomology of a relative Rota-Baxter operator on a Hom-Lie-Yamaguti algebra. In Section 5, we use the cohomological approach to study deformations of relative Rota-Baxter operators.

2. Preliminaries

Throughout this paper, we work on an algebraically closed field $\mathbb{K}$ of characteristics different from 2 and 3. We recall some basic definitions of Hom-Lie-Yamaguti algebra from [10] and [17].

Definition 2.1 ([10]) *A Hom-Lie-Yamaguti algebra is a 4-tuple $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ in which L is a vector space together with a linear map $\alpha : L \to L$, a binary operation $[\cdot,\cdot]$ and a ternary operation$\{\cdot,\cdot,\cdot\}$ on L such that*

$(HLY01)$ $\alpha([x,y]) = [\alpha(x), \alpha(y)]$,

$(HLY02)$ $\alpha(\{x,y,z\}) = \{\alpha(x), \alpha(y), \alpha(z)\}$,

$(HLY03)$ $[x,y] = -[y,x]$,

$(HLY04)$ $\{x,y,z\} = -\{y,x,z\}$,

$(HLY05)$ $([[x,y],\alpha(z)] + \{x,y,z\}) + c.p. = 0$,

$(HLY06)$ $\{[x,y],\alpha(z),\alpha(a)\} + \{[z,x],\alpha(y),\alpha(a)\} + \{[y,z],\alpha(x),\alpha(a)\} = 0$,

$(HLY07)$ $\{\alpha(a),\alpha(b),[x,y]\} = [\{a,b,x\},\alpha^2(y)] + [\alpha^2(x),\{a,b,y\}]$,

$(HLY08)$ $\{\alpha^2(a),\alpha^2(b),\{x,y,z\}\} = \{\{a,b,x\},\alpha^2(y),\alpha^2(z)\} + \{\alpha^2(x),\{a,b,y\},\alpha^2(z)\} + \{\alpha^2(x),\alpha^2(y),\{a,b,z\}\}$,

for all $x,y,z,a,b \in L$ and where c.p. denotes the sum over cyclic permutation of x,y,z, that is $([[x,y],\alpha(z)]+\{x,y,z\})+c.p. = ([[x,y],\alpha(z)]+\{x,y,z\})+([[z,x],\alpha(y)]+\{z,x,y\})+([[y,z],\alpha(x)]+\{y,z,x\})$. In particular, the Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ is said to be regular (reps., involutive), if α is nondegenerate (resp., $\alpha^2 = Id_L$).

A homomorphism between two Hom-Lie-Yamaguti algebras $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ and $(L', [\cdot,\cdot]', \{\cdot,\cdot,\cdot\}', \alpha')$ is a linear map $\varphi : L \to L'$ satisfying $\varphi \circ \alpha = \alpha' \circ \varphi$ and

$$\varphi([x,y]) = [\varphi(x), \varphi(y)]', \ \ \varphi(\{x,y,z\}) = \{\varphi(x), \varphi(y), \varphi(z)\}', \quad \forall\, x,y,z \in L.$$

In particular, if φ is nondegenerate, then φ is called an isomorphism from L to L'.

Example 2.2 Given a Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\})$ with a Lie-Yamaguti algebra homomorphism $\alpha : L \to L$, we can define a Hom-Lie-Yamaguti algebra as the 4-tuple $(L, \alpha\circ[\cdot,\cdot], \alpha\circ$

$\{\cdot,\cdot,\cdot\},\alpha)$.

Example 2.3 Let L be a 3-dimensional Lie-Yamaguti algebra with a basis u_1, u_2 and u_3 defined by $[u_1,u_3]=-u_1$, $[u_2,u_3]=2u_2$ and $\{u_3,u_2,u_3\}=u_2$. The linear transformation α on L is defined by $\alpha(u_1)=u_1$, $\alpha(u_2)=u_2$ and $\alpha(u_3)=u_2+u_3$. Then $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ is a Hom-Lie-Yamaguti algebra.

Example 2.4 Let L be a 4-dimensional Lie-Yamaguti algebra with a basis u_1, u_2, u_3 and u_4 defined by $[u_1,u_2]=2u_4$ and $\{u_1,u_2,u_1\}=u_4$. The linear transformation α on L is defined by $\alpha(u_1)=u_1$, $\alpha(u_2)=ku_2$, $\alpha(u_3)=k_1u_1+k_2u_2+k_3u_3+k_4u_4$ and $\alpha(u_4)=ku_4$ for all $k,k_1,k_2,k_3,k_4\in\mathbb{K}$. Then $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ is a Hom-Lie-Yamaguti algebra.

Definition 2.5 ([17]) *Let $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ be a Hom-Lie-Yamaguti algebra and (V,β) be a Hom-vector space. A representation of $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ on (V,β) consists of a linear map $\rho:L\to\mathrm{End}(V)$ and two bilinear maps $D,\theta:L\times L\to\mathrm{End}(V)$ such that*

$(HR01)$ $\rho(\alpha(x))\circ\beta=\beta\circ\rho(x)$,

$(HR02)$ $D(\alpha(x),\alpha(y))\circ\beta=\beta\circ D(x,y)$,

$(HR03)$ $\theta(\alpha(x),\alpha(y))\circ\beta=\beta\circ\theta(x,y)$,

$(HR04)$ $D(x,y)-\theta(y,x)+\theta(x,y)+\rho([x,y])\circ\beta-\rho(\alpha(x))\rho(y)+\rho(\alpha(y))\rho(x)=0$,

$(HR05)$ $D([x,y],\alpha(z))+D([y,z],\alpha(x))+D([z,x],\alpha(y))=0$,

$(HR06)$ $\theta([x,y],\alpha(a))\circ\beta=\theta(\alpha(x),\alpha(a))\rho(y)-\theta(\alpha(y),\alpha(a))\rho(x)$,

$(HR07)$ $D(\alpha(a),\alpha(b))\rho(x)=\rho(\alpha^2(x))D(a,b)+\rho(\{a,b,x\})\circ\beta^2$,

$(HR08)$ $\theta(\alpha(x),[a,b])\circ\beta=\rho(\alpha^2(a))\theta(x,b)-\rho(\alpha^2(b))\theta(x,a)$,

$(HR09)$ $D(\alpha^2(a),\alpha^2(b))\theta(x,y)=\theta(\alpha^2(x),\alpha^2(y))D(a,b)+\theta(\{a,b,x\},\alpha^2(y))\circ\beta^2+\theta(\alpha^2(x),\{a,b,y\})\circ\beta^2$,

$(HR10)$ $\theta(\alpha^2(a),\{x,y,z\})\circ\beta^2=\theta(\alpha^2(y),\alpha^2(z))\theta(a,x)-\theta(\alpha^2(x),\alpha^2(z))\theta(a,y)+D(\alpha^2(x),\alpha^2(y))\theta(a,z)$,

for all $x,y,z,a,b\in L$. In this case, we also call V a T-module.

It can be concluded from (HR09) that

$$(HR09)'\ D(\alpha^2(a),\alpha^2(b))D(x,y)=D(\alpha^2(x),\alpha^2(y))D(a,b)+D(\{a,b,x\},\alpha^2(y))\circ\beta^2+D(\alpha^2(x),\{a,b,y\})\circ\beta^2.$$

Example 2.6 For any integer $s\geq 0$, we can define the α^s-adjoint representation of a Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ on L as follows

$$\mathrm{ad}_s(x)(z):=[\alpha^s(x),z],\mathcal{L}_s(x,y)(z):=\{\alpha^s(x),\alpha^s(y),z\},\mathcal{R}_s(x,y)(z):=\{z,\alpha^s(x),\alpha^s(y)\},$$

for all $x,y,z\in L$. Let us denote the α^s-adjoint representation of the Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ by the 5-tuple $(L;\alpha,\mathrm{ad}_s,\mathcal{L}_s,\mathcal{R}_s)$. Also we denote $\mathrm{ad}_0(x)$, $\mathcal{L}_0(x,y)$ and $\mathcal{R}_0(x,y)$ simply by $\mathrm{ad}(x)$, $\mathcal{L}(x,y)$ and $\mathcal{R}(x,y)$ for $x,y\in L$.

Proposition 2.7 ([17]) *Let $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ be a Hom-Lie-Yamaguti algebra and (V, β) be a Hom-vector space. Assume that we have a map ρ from L to $\mathrm{End}(V)$ and maps $D, \theta : L \times L \to \mathrm{End}(V)$ satisfying (HR01)-(HR10). Then (ρ, D, θ) is a representation of $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ on (V, β) if and only if $L \oplus V$ is a Hom-Lie-Yamaguti algebra under the following maps:*

$$
\begin{aligned}
&(\alpha+\beta)(x+u) := \alpha(x) + \beta(u),\\
&[x+u, y+v]_{\ltimes} := [x, y] + \rho(x)(v) - \rho(y)(u),\\
&\{x+u, y+v, z+w\}_{\ltimes} := \{x, y, z\} + D(x, y)(w) - \theta(x, z)(v) + \theta(y, z)(u),
\end{aligned}
$$

for all $x, y, z \in L$ and $u, v, w \in V$. In the case, the Hom-Lie-Yamaguti algebra $L \oplus V$ is called a semidirect product of L and V, denoted by $L \ltimes V = (L \oplus V, [\cdot,\cdot]_{\ltimes}, \{\cdot,\cdot,\cdot\}_{\ltimes}, \alpha + \beta)$.

3. Relative Rota-Baxter operators on Hom-Lie-Yamaguti algebras

In this section, we define the notions of s-Rota-Baxter operators and relative Rota-Baxter operators on Hom-Lie-Yamaguti algebras. Moreover, we give some characteristics of the relative Rota-Baxter operator in terms of Nijenhuis operators and graphs.

Definition 3.1 *Let $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ be a Hom-Lie-Yamaguti algebra and s be a non-negative integer. Then, the linear operator $R : L \to L$ is called an s-Rota-Baxter operator on $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ if the following equations hold:*

$$R \circ \alpha = \alpha \circ R,$$

$$[Rx, Ry] = R([\alpha^s R(x), y] + [x, \alpha^s R(y)]), \tag{3.1}$$

$$\{Rx, Ry, Rz\} = R(\{\alpha^s R(x), \alpha^s R(y), z\} + \{x, \alpha^s R(y), \alpha^s R(z)\} - \{y, \alpha^s R(x), \alpha^s R(z)\}), \tag{3.2}$$

for all $x, y, z \in L$.

For $\alpha = Id$, Definition 3.1 coincides with the notion of Rota-Baxter operators on a Lie-Yamaguti algebra.

Definition 3.2 *Let $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ be a Hom-Lie-Yamaguti algebra and $(V; \beta, \rho, D, \theta)$ be a Hom-Lie-Yamaguti algebra representation. The linear operator $R : V \to L$ is called a relative Rota-Baxter operator on $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to the representation $(V; \beta, \rho, D, \theta)$ if the following equations hold:*

$$R \circ \beta = \alpha \circ R,$$

$$[Ru, Rv] = R(\rho(Ru)v - \rho(Rv)u), \tag{3.3}$$

$$\{Ru, Rv, Rw\} = R(D(Ru, Rv)w + \theta(Rv, Rw)u - \theta(Ru, Rw)v), \tag{3.4}$$

for all $u, v, w \in V$.

Remark 3.3 Let us recall from Example 2.6, the α^s -adjoint representation $(L; \alpha, \mathrm{ad}_s, \mathcal{L}_s, \mathcal{R}_s)$ of a Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ for any integer $s \geq 0$. Then, the s-Rota-Baxter operator on the Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ is a relative Rota-Baxter operator on $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(L;\alpha,\mathrm{ad}_s,\mathcal{L}_s,\mathcal{R}_s)$.

Example 3.4 Let $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ be a 2-dimensional Hom-Lie-Yamaguti algebra L with the basis u_1,u_2 defined by

$$[u_1,u_2]=u_1,\{u_1,u_2,u_2\}=u_1,\alpha(u_1)=k_1u_1,\alpha(u_2)=u_2,k_1\in\mathbb{K}.$$

Then the operator $R=\left(\begin{smallmatrix}0&0\\0&k_2\end{smallmatrix}\right)$ is an s-Rota-Baxter operator on L.

The following proposition gives a characterization of a relative Rota-Baxter operator R in terms of a Hom-Lie-Yamaguti subalgebra structure on the graph of R.

Proposition 3.5 *The linear operator $R:V\to L$ is a relative Rota-Baxter operator on $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$ if and only if the graph of the operator R*

$$Gr(R)=\{R(u)+u|u\in V\}$$

is a Hom-Lie-Yamaguti subalgebra of semi-direct product Hom-Lie-Yamaguti algebra

$$L\ltimes V=(L\oplus V,[\cdot,\cdot]_\ltimes,\{\cdot,\cdot,\cdot\}_\ltimes,\alpha+\beta).$$

Proof Let $R:V\to L$ be a linear operator. Then, for all $u,v,w\in V$, we have

$$(\alpha+\beta)(Ru+u)=\alpha\circ R(u)+\beta(u),$$
$$[Ru+u,Rv+v]_\ltimes=[Ru,Rv]+\rho(Ru)v-\rho(Rv)u,$$
$$\{Ru+u,Rv+v,Rw+w)\}_\ltimes=\{Ru,Rv,Rw\}+D(Ru,Rv)w-\theta(Ru,Rw)v+\theta(Rv,Rw)u,$$

which implies that the graph $Gr(R)$ is a Hom-Lie-Yamaguti subalgebra of $L\ltimes V$ if and only if R satisfies Equations $\alpha\circ R=R\circ\beta$, (3.3) and (3.4), which means that R is a relative Rota-Baxter operator. □

Since V and $Gr(R)$ are isomorphic as vector spaces, we get the following result immediately.

Corollary 3.6 *Let $R:V\to L$ be a relative Rota-Baxter operator on Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$. Then $(V,[\cdot,\cdot]_R,\{\cdot,\cdot,\cdot\}_R,\beta)$ is a Hom-Lie-Yamaguti algebra, called the descendent Hom-Lie-Yamaguti algebra of R, where*

$$[u,v]_R=\rho(Ru)v-\rho(Rv)u,$$
$$\{u,v,w\}_R=D(Ru,Rv)w-\theta(Ru,Rw)v+\theta(Rv,Rw)u,$$

for all $u,v,w\in V$. Moreover, R is a homomorphism $(V,[\cdot,\cdot]_R,\{\cdot,\cdot,\cdot\}_R,\beta)$ from $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$.

The following proposition shows that the relative Rota-Baxter operator of Hom-Lie-Yamaguti algebra is equivalent to the 0-Rota-Baxter operator of semi-direct product Hom-Lie-Yamaguti algebra.

Proposition 3.7 *The linear operator $R:V\to L$ is a relative Rota-Baxter operator on $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$ if and only if the operator*

$$R_\ltimes=\begin{pmatrix}0&R\\0&0\end{pmatrix}:L\oplus V\longrightarrow L\oplus V$$

is a 0-Rota-Baxter operators of semi-direct product Hom-Lie -Yamaguti algebra

$$L \ltimes V = (L \oplus V, [\cdot,\cdot]_{\ltimes}, \{\cdot,\cdot,\cdot\}_{\ltimes}, \alpha+\beta).$$

Proof Let us consider the following expressions, where we use definition of the map $\alpha+\beta$, $R_{\ltimes}$ and the brackets $[\cdot,\cdot]_{\ltimes}$, $\{\cdot,\cdot,\cdot\}_{\ltimes}$.

$$(\alpha+\beta)\circ R_{\ltimes}(x+u) = (\alpha+\beta)(Ru+0) = \alpha(Ru)+\beta(0) = \alpha\circ R(u), \tag{3.5}$$

$$R_{\ltimes}\circ(\alpha+\beta)(x+u) = R_{\ltimes}(\alpha(x)+\beta(u)) = R\circ\beta(u) \tag{3.6}$$

and

$$\begin{aligned}
&[R_{\ltimes}(x+u), R_{\ltimes}(y+v)]_{\ltimes} - R_{\ltimes}([(\alpha+\beta)^0 R_{\ltimes}(x+u), y+v]_{\ltimes} + [x+u, (\alpha+\beta)^0 R_{\ltimes}(y+v)]_{\ltimes})\\
&\quad = [Ru+0, Rv+0]_{\ltimes} - R_{\ltimes}([R(u)+0, y+v]_{\ltimes} + [x+u, R(v)+0]_{\ltimes})\\
&\quad = [Ru, Rv] - R_{\ltimes}([Ru, y] + \rho(Ru)v + ([x, Rv] - \rho(Rv)u))\\
&\quad = [Ru, Rv] - R(\rho(Ru)v - \rho(Rv)u),
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
&\{R_{\ltimes}(x+u), R_{\ltimes}(y+v), R_{\ltimes}(z+w)\}_{\ltimes} - R_{\ltimes}(\{(\alpha+\beta)^0 R_{\ltimes}(x+u), (\alpha+\beta)^0 R_{\ltimes}(y+v), z+w\}_{\ltimes} +\\
&\qquad \{x+u, (\alpha+\beta)^0 R_{\ltimes}(y+v), (\alpha+\beta)^0 R_{\ltimes}(z+w)\}_{\ltimes} -\\
&\qquad \{y+v, (\alpha+\beta)^0 R_{\ltimes}(x+u), (\alpha+\beta)^0 R_{\ltimes}(z+w)\}_{\ltimes})\\
&\quad = \{Ru+0, Rv+0, Rw+0\}_{\ltimes} - R_{\ltimes}(\{Ru+0, Rv+0, z+w\}_{\ltimes} + \{x+u, Rv+0, Rw+0\}_{\ltimes} -\\
&\qquad \{y+v, Ru+0, Rw+0\}_{\ltimes})\\
&\quad = \{Ru, Rv, Rw\} - R_{\ltimes}(\{Ru, Rv, z\} + D(Ru, Rv)w + \{x, Rv, Rw\} + \theta(Rv, Rw)u -\\
&\qquad \{y, Ru, Rw\} - \theta(Ru, Rw)v)\\
&\quad = \{Ru, Rv, Rw\} - R(D(Ru, Rv)w + \theta(Rv, Rw)u - \theta(Ru, Rw)v),
\end{aligned} \tag{3.8}$$

for all $x, y, z \in L, u, v, w \in V$. By the above Equations (3.5) and (3.6), it is clear that

$$(\alpha+\beta)\circ R_{\ltimes} = R_{\ltimes}\circ(\alpha+\beta)$$

is equivalent to $\alpha\circ R = R\circ\beta$. By (3.7), Equation (3.1) is equivalent to the Equation (3.3), for $s=0$. By (3.8), Eq. (3.2) is equivalent to the Eq. (3.4), for $s=0$. $\square$

In the sequel, we give the relationship between relative Rota-Baxter operators and Nijenhuis operators. Recall from [18] that the Nijenhuis operator on a Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ is a linear map $N : L \to L$, satisfying

$$\begin{aligned}
&N\circ\alpha = \alpha\circ N,\\
&[Nx, Ny] = N([Nx, y] + [x, Ny] - N[x, y]),\\
&\{Nx, Ny, Nz\} = N(\{Nx, Ny, z\} + \{Nx, y, Nz\} + \{x, Ny, Nz\}) -\\
&\qquad\qquad N^2(\{Nx, y, z\} + \{x, Ny, z\} + \{x, y, Nz\}) + N^3\{x, y, z\},
\end{aligned}$$

for all $x, y, z \in L$.

Corollary 3.8 *The linear operator* $R : V \to L$ *is a relative Rota-Baxter operator on* $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$

with respect to the representation $(V;\beta,\rho,D,\theta)$ *if and only if the operator*

$$N_R=\begin{pmatrix} 0 & R\\ 0 & 0\end{pmatrix}: L\oplus V\longrightarrow L\oplus V$$

is a Nijenhuis operator on a semi-direct product Hom-Lie -Yamaguti algebra $L\ltimes V$.

Proof From $N_R^2=0$, it follows that N_R is a Nijenhuis operator, which is equivalent to that it is a 0-Rota-Baxter operator. □

Now, we use the relative Rota-Baxter operator on Hom-Lie-Yamaguti algebra to characterize the Nijenhuis operator.

Definition 3.9 *Two relative Rota-Baxter operators* $R_1,R_2:V\to L$ *on* $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ *with respect to* $(V;\beta,\rho,D,\theta)$ *are said to be compatible if the sum* $a_1R_1+a_2R_2$ *is a relative Rota-Baxter operator, for any* $a_1,a_2\in\mathbb{K}$. *Equivalently,*

$$\begin{aligned}&[R_1u,R_2v]+[R_2u,R_1v]=R_2(\rho(R_1u)v-\rho(R_1v)u)+R_1(\rho(R_2u)v-\rho(R_2v)u),\\&\{R_iu,R_iv,R_jw\}+\{R_iu,R_jv,R_iw\}+\{R_ju,R_iv,R_iw\}\\&\quad=R_i(D(R_iu,R_jv)w+\theta(R_iv,R_jw)u-\theta(R_iu,R_jw)v)+\\&\qquad R_i(D(R_ju,R_iv)w+\theta(R_jv,R_iw)u-\theta(R_ju,R_iw)v)+\\&\qquad R_j(D(R_iu,R_iv)w+\theta(R_iv,R_iw)u-\theta(R_iu,R_iw)v),\end{aligned}$$

where $i,j\in\{1,2\}$ *and* $i\neq j$, *for all* $u,v,w\in V$.

Proposition 3.10 *Let* $R_1,R_2:V\to L$ *be two compatible relative Rota-Baxter operators in which* R_2 *(resp.,* R_1*) is invertible. Then* $N=R_1\circ R_2^{-1}$ *(resp.,* $N=R_2\circ R_1^{-1}$*) is a Nijenhuis operator on* $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$.

Proof We only prove the case in which R_2 is invertible. Clearly, for any $x,y,z\in L$, there exist $u,v,w\in V$, such that $R_2(u)=x,R_2(v)=y,R_2(w)=z$, then we have

$$\begin{aligned}&N\circ\alpha(x)=(R_1\circ R_2^{-1})\circ\alpha(x)=R_1\circ(R_2^{-1}\circ\alpha)(x)=R_1\circ(\beta\circ R_2^{-1})(x)=R_1\circ\beta(u),\\&\alpha\circ N(x)=\alpha\circ(R_1\circ R_2^{-1})(x)=(\alpha\circ R_1)\circ R_2^{-1}(x)=R_1\circ\beta(u),\end{aligned}$$

thus $N\circ\alpha=\alpha\circ N$, and

$$\begin{aligned}&N([Nx,y]+[x,Ny]-N[x,y])=N([R_1u,R_2v]+[R_2u,R_1v]-N[R_2u,R_2v])\\&\quad=N(R_2(\rho(R_1u)v-\rho(R_1v)u)+R_1(\rho(R_2u)v-\rho(R_2v)u)-NR_2(\rho(R_2u)v-\rho(R_2v)u))\\&\quad=R_1(\rho(R_1u)v-\rho(R_1v)u)=[R_1u,R_1v]=[Nx,Ny],\\&N(\{Nx,Ny,z\}+\{Nx,y,Nz\}+\{x,Ny,Nz\})-N^2(\{Nx,y,z\}+\{x,Ny,z\}+\\&\qquad\{x,y,Nz\})+N^3\{x,y,z\}\\&\quad=N(\{R_1u,R_1v,R_2w\}+\{R_1u,R_2v,R_1w\}+\{R_2u,R_1v,R_1w\})-\\&\qquad N^2(\{R_1u,R_2v,R_2w\}+\{R_2u,R_1v,R_2w\}+\{R_2u,R_2v,R_1w\})+N^3\{R_2u,R_2v,R_2w\}\\&\quad=N(R_1(D(R_1u,R_2v)w+\theta(R_1v,R_2w)u-\theta(R_1u,R_2w)v)+R_1(D(R_2u,R_1v)w+\end{aligned}$$

$$
\begin{aligned}
&\theta(R_2v,R_1w)u-\theta(R_2u,R_1w)v)+R_2(D(R_1u,R_1v)w+\theta(R_1v,R_1w)u-\\
&\theta(R_1u,R_1w)v))-N^2(R_1(D(R_2u,R_2v)w+\theta(R_2v,R_2w)u-\theta(R_2u,R_2w)v)+\\
&R_2(D(R_1u,R_2v)w+\theta(R_1v,R_2w)u-\theta(R_1u,R_2w)v)+R_2(D(R_2u,R_1v)w+\\
&\theta(R_2v,R_1w)u-\theta(R_2u,R_1w)v))+N^3(R_2(D(R_2u,R_2v)w+\theta(R_2v,R_2w)u-\theta(R_2u,R_2w)v))\\
=&NR_1(D(R_1u,R_2v)w+\theta(R_1v,R_2w)u-\theta(R_1u,R_2w)v)+NR_1(D(R_2u,R_1v)w+\\
&\theta(R_2v,R_1w)u-\theta(R_2u,R_1w)v)+R_1(D(R_1u,R_1v)w+\theta(R_1v,R_1w)u-\\
&\theta(R_1u,R_1w)v)-N^2R_1(D(R_2u,R_2v)w+\theta(R_2v,R_2w)u-\theta(R_2u,R_2w)v)-\\
&NR_1(D(R_1u,R_2v)w+\theta(R_1v,R_2w)u-\theta(R_1u,R_2w)v)-NR_1(D(R_2u,R_1v)w+\\
&\theta(R_2v,R_1w)u-\theta(R_2u,R_1w)v)+N^2R_1(D(R_2u,R_2v)w+\theta(R_2v,R_2w)u-\theta(R_2u,R_2w)v)\\
=&R_1(D(R_1u,R_1v)w+\theta(R_1v,R_1w)u-\theta(R_1u,R_1w)v)\\
=&\{R_1u,R_1v,R_1w\}=\{Nx,Ny,Nz\}.
\end{aligned}
$$

We have shown that $N=R_1\circ R_2^{-1}$ is a Nijenhuis operator on $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$. □

4. Cohomology theory

In this section, we define a cohomology of a relative Rota-Baxter operator R on a Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with coefficients in a representation $(V;\beta,\rho,D,\theta)$. Later, we will use this cohomology to study deformation of R.

Let us recall Corollary 3.6: let $R:V\to L$ be a relative Rota-Baxter operator on Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$. Then $(V,[\cdot,\cdot]_R,\{\cdot,\cdot,\cdot\}_R,\beta)$ is a Hom-Lie-Yamaguti algebra. So far, the representation space V has been endowed with a Hom-Lie-Yamaguti algebra structure $([\cdot,\cdot]_R,\{\cdot,\cdot,\cdot\}_R,\beta)$. We will give a representation of V on L (it is viewed as a vector space) in the sequel.

Theorem 4.1 *Let $R:V\to L$ be a relative Rota-Baxter operator on Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$. Then $(L;\alpha,\varrho,\mathfrak{D},\vartheta)$ is a representation of the Hom-Lie-Yamaguti algebra structure $(V,[\cdot,\cdot]_R,\{\cdot,\cdot,\cdot\}_R,\beta)$, where $\varrho:V\to \mathrm{End}(L)$, $\mathfrak{D},\vartheta:V\times V\to \mathrm{End}(L)$, by*

$$
\begin{aligned}
&\varrho(u)x:=[Ru,x]+R(\rho(x)u),\\
&\mathfrak{D}(u,v)x:=\{Ru,Rv,x\}+R(\theta(Ru,x)v-\theta(Rv,x)u),\\
&\vartheta(u,v)x:=\{x,Ru,Rv\}+R(\theta(x,Rv)u-D(x,Ru)v),
\end{aligned}
$$

for all $u,v\in V,x\in L$.

Proof Let $N:L\to L$ be a Nijenhuis operator on a Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$. Then $(L,[\cdot,\cdot]_N,\{\cdot,\cdot,\cdot\}_N,\alpha)$ is a Hom-Lie-Yamaguti algebra, where $[\cdot,\cdot]_N,\{\cdot,\cdot,\cdot\}_N$ are given by

$$[x,y]_N=[Nx,y]+[x,Ny]-N[x,y], \tag{4.1}$$

$$\{x,y,z\}_N=\{Nx,Ny,z\}+\{Nx,y,Nz\}+\{x,Ny,Nz\}-N(\{Nx,y,z\}+\{x,Ny,z\}+$$

$$\{x,y,Nz\}) + N^2\{x,y,z\}, \tag{4.2}$$

for all $x,y,z \in L$. Let $R: V \to L$ be a relative Rota-Baxter operator on Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$. By Corollary 3.8, $N_R = \left(\begin{smallmatrix} 0 & R \\ 0 & 0 \end{smallmatrix}\right): L\oplus V \longrightarrow L\oplus V$ is a Nijenhuis operator on the semi-direct product Hom-Lie-Yamaguti algebra $L\ltimes V$ and $N_R^2 = 0$. Then by Eqs. (4.1) and (4.2), there is a Hom-Lie-Yamaguti algebra structure $([\cdot,\cdot]_{N_R},\{\cdot,\cdot,\cdot\}_{N_R},\beta+\alpha)$ on the vector space $V\oplus L \cong L\oplus V$ by

$$\begin{aligned}
&[u+x,v+y]_{N_R} = [x+u,y+v]_{N_R}\\
&= [N_R(x+u),y+v]_{\ltimes} + [x+u,N_R(y+v)]_{\ltimes} - N_R[x+u,y+v]_{\ltimes}\\
&= [Ru,y+v]_{\ltimes} + [x+u,Rv]_{\ltimes} - N_R[x+u,y+v]_{\ltimes}\\
&= [Ru,y] + \rho(Ru)v + [x,Rv] - \rho(Rv)u - N_R([x+y] + \rho(x)v - \rho(y)u)\\
&= [Ru,y] + \rho(Ru)v + [x,Rv] - \rho(Rv)u - R(\rho(x)v - \rho(y)u)\\
&= \rho(Ru)v - \rho(Rv)u + [Ru,y] + R(\rho(y)u) - ([Rv,x] + R(\rho(x)v))\\
&= [u,v]_R + \varrho(u)y - \varrho(v)x,\\
&\quad \{u+x,v+y,w+z\}_{N_R} = \{x+u,y+v,z+w\}_{N_R}\\
&= \{N_R(x+u),N_R(y+v),z+w\}_{\ltimes} + \{N_R(x+u),y+v,N_R(z+w)\}_{\ltimes}+\\
&\quad \{x+u,N_R(y+v),N_R(z+w)\}_{\ltimes} - N_R(\{N_R(x+u),y+v,z+w\}_{\ltimes}+\\
&\quad \{x+u,N_R(y+v),z+w\}_{\ltimes} + \{x+w,y+v,N_R(z+w)\}_{\ltimes})\\
&= \{Ru,Rv,z+w\}_{\ltimes} + \{Ru,y+v,Rw\}_{\ltimes} + \{x+u,Rv,Rw\}_{\ltimes}-\\
&\quad N_R(\{Ru,y+v,z+w\}_{\ltimes} + \{x+u,Rv,z+w\}_{\ltimes} + \{x+u,y+v,Rw\}_{\ltimes})\\
&= \{Ru,Rv,z\} + D(Ru,Rv)w + \{Ru,y,Rw\} - \theta(Ru,Rw)v + \{x,Rv,Rw\} + \theta(Rv,Rw)u-\\
&\quad N_R(\{Ru,y,z\} + D(Ru,y)w - \theta(Ru,z)v + \{x,Rv,z\} + D(x,Rv)w + \theta(Rv,z)u+\\
&\quad \{x,y,Rw\} - \theta(x,Rw)v + \theta(y,Rw)u)\\
&= \{Ru,Rv,z\} + D(Ru,Rv)w + \{Ru,y,Rw\} - \theta(Ru,Rw)v + \{x,Rv,Rw\} + \theta(Rv,Rw)u-\\
&\quad R(D(Ru,y)w - \theta(Ru,z)v + D(x,Rv)w + \theta(Rv,z)u - \theta(x,Rw)v + \theta(y,Rw)u)\\
&= \big(D(Ru,Rv)w + \theta(Rv,Rw)u - \theta(Ru,Rw)v\big) + \{Ru,Rv,z\} + R(\theta(Ru,z)v - \theta(Rv,z)u)-\\
&\quad \{y,Ru,Rw\} - R(\theta(y,Rw)u - D(y,Ru)w) + \{x,Rv,Rw\} + R(\theta(x,Rw)v - D(x,Rv)w)\\
&= \{u,v,w\}_R + \mathfrak{D}(u,v)z - \vartheta(u,w)y + \vartheta(v,w)x,
\end{aligned}$$

for all $u,v,w \in V, x,y,z \in L$. By Proposition 2.7, we deduce that $(\varrho,\mathfrak{D},\vartheta)$ is a representation of the Hom-Lie-Yamaguti algebra $(V,[\cdot,\cdot]_R,\{\cdot,\cdot,\cdot\}_R,\beta)$ on (L,α). □

Now, we describe the cohomology of a relative Rota-Baxter operator on Hom-Lie-Yamaguti algebra in terms of Hom-Lie-Yamaguti algebra cohomology [17].

Let $(L;\alpha,\varrho,\mathfrak{D},\vartheta)$ be a representation of Hom-Lie-Yamaguti algebra $(V,[\cdot,\cdot]_R,\{\cdot,\cdot,\cdot\}_R,\beta)$. We denote the set of p-cochains by $C^p_R(V,L)$ $(p\geq 1)$, where if $p\geq 2$,

$$C^p_R(V,L) = \{f \in \mathrm{Hom}(V\times V\times\cdots\times V,L) \mid f(\beta(v_1),\ldots,\beta(v_p)) = \alpha\circ f(v_1,\ldots,v_p),$$

$$f(v_1,\dots,v_{2i-1},v_{2i},\dots,v_p)=0 \ \text{ if } v_{2i-1}=v_{2i}\},$$

if $p=1$, $C^1_R(V,L)=\{f\in \mathrm{Hom}(V,L)\mid f(\beta(v_1))=\alpha\circ f(v_1)\}$.

For $n\geq 1$, let $\delta_R: C^{2n}_R(V,L)\times C^{2n+1}_R(V,L)\to C^{2n+2}_R(V,L)\times C^{2n+3}_R(V,L), (f,g)\mapsto(\delta^I_R(f,g),$ $\delta^{II}_R(f,g))$ be the corresponding coboundary operator of V with coefficient in L and it is defined as follows

$$\begin{aligned}
&(\delta^I_R(f,g))(v_1,v_2,\dots,v_{2n+2})\\
&\quad=[R\circ\beta^{2n}(v_{2n+1}),g(v_1,v_2,\dots,v_{2n},v_{2n+2})]+R(\rho(g(v_1,v_2,\dots,v_{2n},v_{2n+2}))\beta^{2n}(v_{2n+1}))-\\
&\qquad[R\circ\beta^{2n}(v_{2n+2}),g(v_1,v_2,\dots,v_{2n},v_{2n+1})]-R(\rho(g(v_1,v_2,\dots,v_{2n},v_{2n+1}))\beta^{2n}(v_{2n+2}))-\\
&\qquad g(\beta(v_1),\beta(v_2),\dots,\beta(v_{2n}),\rho(Rv_{2n+1})v_{2n+2}-\rho(Rv_{2n+2})v_{2n+1})+\\
&\qquad\sum_{k=1}^{n}(-1)^{n+k+1}\{\beta^{2n-1}(v_{2k-1}),\beta^{2n-1}(v_{2k}),f(v_1,\dots,\widehat{v}_{2k-1},\widehat{v}_{2k},\dots,v_{2n+2})\}+\\
&\qquad\sum_{k=1}^{n}(-1)^{n+k+1}R(\theta(R\circ\beta^{2n-1}(v_{2k-1}),f(v_1,\dots,\widehat{v}_{2k-1},\widehat{v}_{2k},\dots,v_{2n+2}))\beta^{2n-1}(v_{2k})-\\
&\qquad\qquad\theta(R\circ\beta^{2n-1}(v_{2k}),f(v_1,\dots,\widehat{v}_{2k-1},\widehat{v}_{2k},\dots,v_{2n+2}))\beta^{2n-1}(v_{2k-1}))+\\
&\qquad\sum_{k=1}^{n}\sum_{j=2k+1}^{2n+2}(-1)^{n+k}f(\beta^2(v_1),\dots,\widehat{v}_{2k-1},\widehat{v}_{2k},\dots,D(Rv_{2k-1},Rv_{2k})v_j+\theta(Rv_{2k},Rv_j)v_{2k-1}-\\
&\qquad\qquad\theta(Rv_{2k-1},Rv_j)v_{2k},\dots,\beta^2(v_{2n+2})),\\
&(\delta^{II}_R(f,g))(v_1,v_2,\dots,v_{2n+3})\\
&\quad=\{g(v_1,\dots,v_{2n+1}),R\circ\beta^{2n}(v_{2n+2}),R\circ\beta^{2n}(v_{2n+3})\}+\\
&\qquad R(\theta(g(v_1,\dots,v_{2n+1}),R\circ\beta^{2n}(v_{2n+3}))\beta^{2n}(v_{2n+2})-\\
&\qquad\qquad D(g(v_1,\dots,v_{2n+1}),R\circ\beta^{2n}(v_{2n+2}))\beta^{2n}(v_{2n+3}))-\\
&\qquad\{g(v_1,\dots,v_{2n},v_{2n+2}),R\circ\beta^{2n}(v_{2n+1}),R\circ\beta^{2n}(v_{2n+3})\}-\\
&\qquad R(\theta(g(v_1,\dots,v_{2n},v_{2n+2}),R\circ\beta^{2n}(v_{2n+3}))\beta^{2n}(v_{2n+1})-\\
&\qquad\qquad D(g(v_1,\dots,v_{2n},v_{2n+2}),R\circ\beta^{2n}(v_{2n+1}))\beta^{2n}(v_{2n+3}))+\\
&\qquad\sum_{k=1}^{n+1}(-1)^{n+k+1}\{R\circ\beta^{2n}(v_{2k-1}),R\circ\beta^{2n}(v_{2k}),g(v_1,\dots,\widehat{v}_{2k-1},\widehat{v}_{2k},\dots,v_{2n+3})\}+\\
&\qquad\sum_{k=1}^{n+1}(-1)^{n+k+1}R(\theta(R\circ\beta^{2n}(v_{2k-1}),g(v_1,\dots,\widehat{v}_{2k-1},\widehat{v}_{2k},\dots,v_{2n+3}))\beta^{2n}(v_{2k})-\\
&\qquad\qquad\theta(R\circ\beta^{2n}(v_{2k}),g(v_1,\dots,\widehat{v}_{2k-1},\widehat{v}_{2k},\dots,v_{2n+3})\beta^{2n}(v_{2k-1}))+\\
&\qquad\sum_{k=1}^{n+1}\sum_{j=2k+1}^{2n+3}(-1)^{n+k}g(\beta^2(v_1),\dots,\widehat{v}_{2k-1},\widehat{v}_{2k},\dots,D(Rv_{2k-1},Rv_{2k})v_j+\theta(Rv_{2k},Rv_j)v_{2k-1}-\\
&\qquad\qquad\theta(Rv_{2k-1},Rv_j)v_{2k},\dots,\beta^2(v_{2n+3})),
\end{aligned}$$

for all $v_1,v_2,\dots,v_{2n+3}\in V$.

For $n\geq 1$, denote the set of $(2n,2n{+}1)$-cocycles and $(2n,2n{+}1)$-coboundaries by $Z^{2n\times 2n+1}_R(V,L)$

and $B_R^{2n\times 2n+1}(V,L)$, respectively. We define the set

$$H_R^{2n\times 2n+1}(V,L)=\frac{Z_R^{2n\times 2n+1}(V,L)}{B_R^{2n\times 2n+1}(V,L)}$$

as a $(2n,2n+1)$-th cohomology group for the relative Rota-Baxter operator R on the Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$.

In particular, for $n=0$, a linear map $g\in C_R^1(V,L)$ is called $(0,1)$-cocycle if

$$R(\rho(g(v_1))v_2-\rho(g(v_2))v_1)=[g(v_1),Rv_2]+[Rv_1,g(v_2)]-g(\rho(Rv_1)v_2-\rho(Rv_2)v_1), \tag{4.3}$$

$$\begin{aligned}&R(\theta(Rv_2,g(v_3))v_1+\theta(g(v_2),Rv_3)v_1-\theta(Rv_1,g(v_3))v_2-\theta(g(v_1),Rv_3)v_2+\\&\quad D(Rv_1,g(v_2))v_3+D(g(v_1),Rv_2)v_3)\\&=\{Rv_1,Rv_2,g(v_3)\}+\{Rv_1,g(v_2),Rv_3\}+\{g(v_1),Rv_2,Rv_3\}-\\&\quad g(D(Rv_1,Rv_2)v_3+\theta(Rv_2,Rv_3)v_1-\theta(Rv_1,Rv_3)v_2),\end{aligned} \tag{4.4}$$

for all $v_1,v_2,v_3\in V$. The space of $(0,1)$-cocycles is denoted by $Z_R^{0\times 1}(V,L)$. $H_R^{0\times 1}(V,L)=Z_R^{0\times 1}(V,L)$ is a $(0,1)$-th cohomology group for the relative Rota-Baxter operator R.

Proposition 4.2 *Let R be a relative Rota-Baxter operator on a Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$. If $\alpha^2=Id_L,\beta^2=Id_V$ and there exist two elements $a,b\in L$ such that $\alpha(a)=a,\alpha(b)=b$, we define $\delta(a,b):V\to L$ by*

$$\delta(a,b)v=R(D(a,b)v)-\{a,b,Rv\},$$

for any $v\in V$. Then $\delta(a,b)$ is a (0,1)-cocycle for the relative Rota-Baxter operator R.

Proof Just verify that $\delta(a,b)$ meets Eqs. (4.3) and (4.4). For any $v_1,v_2,v_3\in V$, we have

$$\begin{aligned}&[\delta(a,b)(v_1),Rv_2]+[Rv_1,\delta(a,b)(v_2)]-\delta(a,b)(\rho(Rv_1)v_2-\rho(Rv_2)v_1)\\&=[R(D(a,b)v_1)-\{a,b,Rv_1\},Rv_2]+[Rv_1,R(D(a,b)v_2)-\{a,b,Rv_2\}]-\\&\quad R(D(a,b)(\rho(Rv_1)v_2-\rho(Rv_2)v_1))+\{a,b,R(\rho(Rv_1)v_2-\rho(Rv_2)v_1)\}\\&=[R(D(a,b)v_1),Rv_2]-[\{a,b,Rv_1\},Rv_2]+[Rv_1,R(D(a,b)v_2)]-[Rv_1,\{a,b,Rv_2\}]-\\&\quad R(D(a,b)(\rho(Rv_1)v_2))+R(D(a,b)(\rho(Rv_2)v_1))+\{a,b,[Rv_1,Rv_2]\}\\&=R(\rho(R(D(a,b)v_1)v_2))-R(\rho(Rv_2)(D(a,b)v_1))+R(\rho(Rv_1)D(a,b)v_2)-\\&\quad R(\rho(R(D(a,b)v_2))v_1)-R(D(a,b)(\rho(Rv_1)v_2))+R(D(a,b)(\rho(Rv_2)v_1))\\&=R(\rho(R(D(a,b)v_1))v_2)-R(\rho(R(D(a,b)v_2))v_1)+R(D(a,b)\rho(Rv_2)v_1-\\&\quad \rho(Rv_2)D(a,b)v_1)-R(D(a,b)\rho(Rv_1)v_2-\rho(Rv_1)D(a,b)v_2)\\&=R(\rho(R(D(a,b)v_1))v_2)-R(\rho(R(D(a,b)v_2))v_1)+R(\rho(\{a,b,Rv_2\})v_1)-R(\rho(\{a,b,Rv_1\})v_2)\\&=R(\rho(R(D(a,b)v_1)-\{a,b,Rv_1\})v_2-\rho(R(D(a,b)v_2)-\{a,b,Rv_2\})v_1)\\&=R(\rho(\delta(a,b)v_1)v_2-\rho(\delta(a,b)v_2)v_1),\end{aligned}$$

$$\begin{aligned}&\{Rv_1,Rv_2,\delta(a,b)(v_3)\}+\{Rv_1,\delta(a,b)(v_2),Rv_3\}+\{\delta(a,b)(v_1),Rv_2,Rv_3\}-\\&\quad \delta(a,b)(D(Rv_1,Rv_2)v_3+\theta(Rv_2,Rv_3)v_1-\theta(Rv_1,Rv_3)v_2)\end{aligned}$$

$$
\begin{aligned}
&= \{Rv_1, Rv_2, R(D(a,b)v_3) - \{a,b,Rv_3\}\} + \{Rv_1, R(D(a,b)v_2) - \{a,b,Rv_2\}, Rv_3\} + \\
&\quad \{R(D(a,b)v_1) - \{a,b,Rv_1\}, Rv_2, Rv_3\} - R(D(a,b)(D(Rv_1,Rv_2)v_3 + \theta(Rv_2,Rv_3)v_1 - \\
&\quad \theta(Rv_1,Rv_3)v_2)) + \{a,b,R(D(Rv_1,Rv_2)v_3 + \theta(Rv_2,Rv_3)v_1 - \theta(Rv_1,Rv_3)v_2)\} \\
&= \{Rv_1, Rv_2, R(D(a,b)v_3)\} - \{Rv_1, Rv_2, \{a,b,Rv_3\}\} + \{Rv_1, R(D(a,b)v_2), Rv_3\} - \\
&\quad \{Rv_1, \{a,b,Rv_2\}, Rv_3\} + \{R(D(a,b)v_1), Rv_2, Rv_3\} - \{\{a,b,Rv_1\}, Rv_2, Rv_3\} - \\
&\quad R(D(a,b)(D(Rv_1,Rv_2)v_3) - R(D(a,b)(\theta(Rv_2,Rv_3)v_1) + R(D(a,b)(\theta(Rv_1,Rv_3)v_2)) + \\
&\quad \{a,b,\{Rv_1,Rv_2,Rv_3\}\} \\
&= \{Rv_1, Rv_2, R(D(a,b)v_3)\} + \{Rv_1, R(D(a,b)v_2), Rv_3\} + \{R(D(a,b)v_1), Rv_2, Rv_3\} - \\
&\quad R(D(a,b)D(Rv_1,Rv_2)v_3) - R(D(a,b)\theta(Rv_2,Rv_3)v_1) + R(D(a,b)\theta(Rv_1,Rv_3)v_2) \\
&= R(D(Rv_1,Rv_2)D(a,b)v_3 + \theta(Rv_2, R(D(a,b)v_3))v_1 - \theta(Rv_1, R(D(a,b)v_3))v_2 + \\
&\quad D(Rv_1, R(D(a,b)v_2))v_3 + \theta(R(D(a,b)v_2), Rv_3)v_1 - \theta(Rv_1.Rv_3)D(a,b)v_2 + \\
&\quad D(R(D(a,b)v_1), Rv_2)v_3 + \theta(Rv_2,Rv_3)D(a,b)v_1 - \theta(R(D(a,b)v_1), Rv_3)v_2 - \\
&\quad D(a,b)D(Rv_1,Rv_2)v_3 - D(a,b)\theta(Rv_2,Rv_3)v_1 + D(a,b)\theta(Rv_1,Rv_3)v_2) \\
&= R(\theta(Rv_2, R(D(a,b)v_3))v_1 - \theta(Rv_1, R(D(a,b)v_3))v_2 - D(a,b)D(Rv_1,Rv_2)v_3 + \\
&\quad D(Rv_1,Rv_2)D(a,b)v_3 + D(a,b)\theta(Rv_1,Rv_3)v_2 - \theta(Rv_1.Rv_3)D(a,b)v_2 - \\
&\quad D(a,b)\theta(Rv_2,Rv_3)v_1 + \theta(Rv_2,Rv_3)D(a,b)v_1 - \theta(R(D(a,b)v_1), Rv_3)v_2 + \\
&\quad D(R(D(a,b)v_1), Rv_2)v_3 + \theta(R(D(a,b)v_2), Rv_3)v_1 + D(Rv_1, R(D(a,b)v_2))v_3) \\
&= R(\theta(Rv_2, R(D(a,b)v_3))v_1 - \theta(Rv_1, R(D(a,b)v_3))v_2 - D(Rv_1, \{a,b,Rv_2\})v_3 - \\
&\quad D(\{a,b,Rv_1\}, Rv_2)v_3 + \theta(Rv_1, \{a,b,Rv_3\})v_2 + \theta(\{a,b,Rv_1\}, Rv_3)v_2 - \\
&\quad \theta(Rv_2, \{a,b,Rv_3\})v_1 - \theta(\{a,b,Rv_2\}, Rv_3)v_1 - \theta(R(D(a,b)v_1), Rv_3)v_2 + \\
&\quad D(R(D(a,b)v_1), Rv_2)v_3 + \theta(R(D(a,b)v_2), Rv_3)v_1 + D(Rv_1, R(D(a,b)v_2))v_3) \\
&= R(\theta(Rv_2, R(D(a,b)v_3))v_1 - \theta(Rv_2, \{a,b,Rv_3\})v_1 - \theta(Rv_1, R(D(a,b)v_3))v_2 + \\
&\quad \theta(Rv_1, \{a,b,Rv_3\})v_2 + \theta(R(D(a,b)v_2), Rv_3)v_1 - \theta(\{a,b,Rv_2\}, Rv_3)v_1 - \\
&\quad \theta(R(D(a,b)v_1), Rv_3)v_2 + \theta(\{a,b,Rv_1\}, Rv_3)v_2 + D(Rv_1, R(D(a,b)v_2))v_3 - \\
&\quad D(Rv_1, \{a,b,Rv_2\})v_3 + D(R(D(a,b)v_1), Rv_2)v_3 - D(\{a,b,Rv_1\}, Rv_2)v_3) \\
&= R(\theta(Rv_2, R(D(a,b)v_3) - \{a,b,Rv_3\})v_1 - \theta(Rv_1, R(D(a,b)v_3) - \{a,b,Rv_3\})v_2 + \\
&\quad \theta(R(D(a,b)v_2) - \{a,b,Rv_2\}, Rv_3)v_1 - \theta(R(D(a,b)v_1) - \{a,b,Rv_1\}, Rv_3)v_2 + \\
&\quad D(Rv_1, R(D(a,b)v_2) - \{a,b,Rv_2\})v_3 + D(R(D(a,b)v_1) - \{a,b,Rv_1\}, Rv_2)v_3) \\
&= R(\theta(Rv_2, \delta(a,b)(v_3))v_1 - \theta(Rv_1, \delta(a,b)(v_3))v_2 + \theta(\delta(a,b)(v_2), Rv_3)v_1 - \\
&\quad \theta(\delta(a,b)(v_1), Rv_3)v_2 + D(Rv_1, \delta(a,b)(v_2))v_3 + D(\delta(a,b)(v_1), Rv_2)v_3).
\end{aligned}
$$

This completes the proof. □

5. Deformation theory

In this section, we will use the cohomology theory constructed in the former section to characterize deformations of relative Rota-Baxter operators on Hom-Lie-Yamaguti algebras.

Definition 5.1 *Let R be a relative Rota-Baxter operator on a Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$, and let $\mathfrak{I}: V \to L$ be a linear map. If $R_t = R + t\mathfrak{I}$ is still a relative Rota-Baxter operator on $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to a representation $(V;\beta,\rho,D,\theta)$ for all t, we say that $\mathfrak{I}$ generates a linear deformation of the relative Rota-Baxter operator R.*

Suppose $\mathfrak{I}$ generates a linear deformation of the relative Rota-Baxter operator R, then we have

$$\begin{aligned}
&R_t \circ \beta = \alpha \circ R_t,\\
&[R_t u, R_t v] = R_t(\rho(R_t u)v - \rho(R_t v)u),\\
&\{R_t u, R_t v, R_t w\} = R_t(D(R_t u, R_t v)w + \theta(R_t v, R_t w)u - \theta(R_t u, R_t w)v),
\end{aligned}$$

for all $u, v, w \in V$. This is equivalent to the following conditions

$$\mathfrak{I} \circ \beta = \alpha \circ \mathfrak{I}, \tag{5.1}$$

$$[Ru, \mathfrak{I}v] + [\mathfrak{I}u, Rv] = \mathfrak{I}(\rho(Ru)v - \rho(Rv)u) + R(\rho(\mathfrak{I}u)v - \rho(\mathfrak{I}v)u), \tag{5.2}$$

$$[\mathfrak{I}u, \mathfrak{I}v] = \mathfrak{I}(\rho(\mathfrak{I}u)v - \rho(\mathfrak{I}v)u), \tag{5.3}$$

$$\begin{aligned}
&\{Ru, Rv, \mathfrak{I}w\} + \{Ru, \mathfrak{I}v, Rw\} + \{\mathfrak{I}u, Rv, Rw\}\\
&\quad = R(D(Ru, \mathfrak{I}v)w + \theta(Rv, \mathfrak{I}w)u - \theta(Ru, \mathfrak{I}w)v) + R(D(\mathfrak{I}u, Rv)w + \theta(\mathfrak{I}v, Rw)u -\\
&\qquad \theta(\mathfrak{I}u, Rw)v) + \mathfrak{I}(D(Ru, Rv)w + \theta(Rv, Rw)u - \theta(Ru, Rw)v),
\end{aligned} \tag{5.4}$$

$$\begin{aligned}
&\{\mathfrak{I}u, \mathfrak{I}v, Rw\} + \{\mathfrak{I}u, Rv, \mathfrak{I}w\} + \{Ru, \mathfrak{I}v, \mathfrak{I}w\}\\
&\quad = \mathfrak{I}(D(\mathfrak{I}u, Rv)w + \theta(\mathfrak{I}v, Rw)u - \theta(\mathfrak{I}u, Rw)v) + \mathfrak{I}(D(Ru, \mathfrak{I}v)w + \theta(Rv, \mathfrak{I}w)u -\\
&\qquad \theta(Ru, \mathfrak{I}w)v) + R(D(\mathfrak{I}u, \mathfrak{I}v)w + \theta(\mathfrak{I}v, \mathfrak{I}w)u - \theta(\mathfrak{I}u, \mathfrak{I}w)v),
\end{aligned} \tag{5.5}$$

$$\{\mathfrak{I}u, \mathfrak{I}v, \mathfrak{I}w\} = \mathfrak{I}(D(\mathfrak{I}u, \mathfrak{I}v)w + \theta(\mathfrak{I}v, \mathfrak{I}w)u - \theta(\mathfrak{I}u, \mathfrak{I}w)v), \tag{5.6}$$

for all $u, v, w \in V$. Thus, R_t is a linear deformation of R if and only if Eqs. (5.1)–(5.6) hold. From (5.1), (5.3) and (5.6) it follows that the map $\mathfrak{I}$ is a relative Rota-Baxter operator on the Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$.

Proposition 5.2 *Let $R_t = R + t\mathfrak{I}$ be a linear deformation of a relative Rota-Baxter operator R on a Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$. Then $\mathfrak{I} \in C^1_R(V, L)$ is a (0,1)-cocycle of the relative Rota-Baxter operator R. Moreover, the $(0,1)$-cocycle $\mathfrak{I}$ is called the infinitesimal of the linear deformation R_t of R.*

Proof Observe that Eqs. (5.1), (5.2) and (5.4) imply that $\mathfrak{I} \in C^1_R(V, L)$ is a (0,1)-cocycle. □

Definition 5.3 *Let R and R' be two relative Rota-Baxter operators on a Hom-Lie-Yamaguti* *algebra* $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ *with respect to the representation* $(V; \beta, \rho, D, \theta)$*. A homomorphism from* R' *to* R *is a pair* (φ_L, φ_V)*, where* $\varphi_L : L \to L$ *is a Hom-Lie-Yamaguti algebra homomorphism and* $\varphi_V : V \to V$ *is a linear map satisfying*

$$R \circ \varphi_V = \varphi_L \circ R', \quad \beta \circ \varphi_V = \varphi_V \circ \beta, \quad \varphi_V(\rho(x)v) = \rho(\varphi_L(x))\varphi_V(v),$$
$$\varphi_V(D(x,y)v) = D(\varphi_L(x), \varphi_L(y))\varphi_V(v), \quad \varphi_V(\theta(x,y)v) = \theta(\varphi_L(x), \varphi_L(y))\varphi_V(v),$$

for all $x, y \in L, v \in V$*. In particular, if* φ_L *and* φ_V *are invertible, then* (φ_L, φ_V) *is called an isomorphism from* R' *to* R*.*

Definition 5.4 *Let* R *be a relative Rota-Baxter operator on an involutive Hom-Lie-Yamaguti algebra* $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ *with respect to the representation* $(V; \beta, \rho, D, \theta)$*. Two linear deformations* $R_t^1 = R + t\mathfrak{I}_1$ *and* $R_t^2 = R + t\mathfrak{I}_2$ *are said to be equivalent if there exist two elements* $a, b \in L$ *such that* $\alpha(a) = a, \alpha(b) = b$ *and the pair* $(Id_L + t\mathcal{L}(a,b), Id_V + tD(a,b))$ *is a homomorphism from* R_t^2 *to* R_t^1*.*

Let $(Id_L + t\mathcal{L}(a,b), Id_V + tD(a,b))$ be a homomorphism from R_t^2 to R_t^1. Then $Id_L + t\mathcal{L}(a,b)$ is an endomorphism of L, we have

$$[\{a,b,x\}, \{a,b,y\}] = 0,$$
$$\{\{a,b,x\}, \{a,b,y\}, z\} + \{\{a,b,x\}, y, \{a,b,z\}\} + \{x, \{a,b,y\}, \{a,b,z\}\} = 0,$$
$$\{\{a,b,x\}, \{a,b,y\}, \{a,b,z\}\} = 0,$$

for all $x, y, z \in L$.

By $R_t^1(Id_V + tD(a,b))v) = (Id_L + t\mathcal{L}(a,b))R_t^2(v)$, we have

$$(\mathfrak{I}_2 - \mathfrak{I}_1)(v) = R(D(a,b)v) - \{a, b, Rv\}, \tag{5.7}$$
$$\mathfrak{I}_1(D(a,b)v) = \{a, b, \mathfrak{I}_2(v)\}.$$

By $\beta \circ (Id_V + tD(a,b)) = (Id_V + tD(a,b)) \circ \beta$, we have $\beta \circ D(a,b) = D(a,b) \circ \beta$.

By $(Id_V + tD(a,b))(\rho(x)v) = \rho((Id_L + t\mathcal{L}(a,b))(x))(Id_V + tD(a,b))v$, we have

$$\rho(\{a,b,x\})D(a,b) = 0, \quad \beta^2 = Id_V.$$

By $(Id_V + tD(a,b))(D(z,w)v) = D((Id_L + t\mathcal{L}(a,b))z, (Id_L + t\mathcal{L}(a,b))w)(Id_V + tD(a,b))v$, we have

$$D(z, \{a,b,w\})D(a,b) + D(\{a,b,z\}, w)D(a,b) + D(\{a,b,z\}, \{a,b,w\}) = 0,$$
$$D(\{a,b,z\}, \{a,b,w\})D(a,b) = 0.$$

By $(Id_V + tD(a,b))(\theta(z,w)v) = \theta((Id_L + t\mathcal{L}(a,b))z, (Id_L + t\mathcal{L}(a,b))w)(Id_V + tD(a,b))v$, we have

$$\theta(z, \{a,b,w\})D(a,b) + \theta(\{a,b,z\}, w)D(a,b) + \theta(\{a,b,z\}, \{a,b,w\}) = 0,$$
$$\theta(\{a,b,z\}, \{a,b,w\})D(a,b) = 0.$$

Note that Eq. (5.7) means that $\mathfrak{I}_2 - \mathfrak{I}_1 = \delta(a,b)$. By Proposition 4.2, we have the following result.

Theorem 5.5 *Let R be a relative Rota-Baxter operator on an involutive Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$. If two linear deformations $R_t^1=R+t\mathfrak{I}_1$ and $R_t^2=R+t\mathfrak{I}_2$ of R are equivalent, then $\mathfrak{I}_1$ and $\mathfrak{I}_2$ define the same cohomology class in $H_R^{0\times 1}(V,L)$.*

Next, we deal with the formal deformations of relative Rota-Baxter operators on Hom-Lie-Yamaguti algebras. Let $\mathbb{K}[[t]]$ be a ring of power series of one variable t, and let $L[[t]]$ be the set of formal power series over L. If $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ is a Hom-Lie-Yamaguti algebra, then there is a Hom-Lie-Yamaguti algebra structure over the ring $\mathbb{K}[[t]]$ on $L[[t]]$ given by

$$\alpha_t=\sum_{i=0}^{\infty}\alpha_i t^i,\alpha_i\in\mathrm{End}(L),\alpha_0=\alpha,$$
$$\Big[\sum_{i=0}^{\infty}x_it^i,\sum_{i=0}^{\infty}y_jt^j\Big]=\sum_{s=0}^{\infty}\sum_{i+j=s}[x_i,y_j]t^s,$$
$$\Big\{\sum_{i=0}^{\infty}x_it^i,\sum_{i=0}^{\infty}y_jt^j,\sum_{k=0}^{\infty}z_kt^k\Big\}=\sum_{s=0}^{\infty}\sum_{i+j+k=s}\{x_i,y_j,z_k\}t^s.$$

For any representation $(V;\beta,\rho,D,\theta)$ of a Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$, there is a nature representation of the Hom-Lie-Yamaguti algebra $L[[t]]$ on the $\mathbb{K}[[t]]$-module $V[[t]]$, which is given by

$$\beta_t=\sum_{i=0}^{\infty}\beta_it^i,\beta_i\in\mathrm{End}(V),\beta_0=\beta,\quad \rho\Big(\sum_{i=0}^{\infty}x_it^i\Big)\Big(\sum_{i=0}^{\infty}v_jt^j\Big)=\sum_{s=0}^{\infty}\sum_{i+j=s}\rho(x_i)v_jt^s,$$
$$D\Big(\sum_{i=0}^{\infty}x_it^i,\sum_{i=0}^{\infty}y_jt^j\Big)\Big(\sum_{k=0}^{\infty}v_kt^k\Big)=\sum_{s=0}^{\infty}\sum_{i+j+k=s}D(x_i,y_j)v_kt^s,$$
$$\theta\Big(\sum_{i=0}^{\infty}x_it^i,\sum_{i=0}^{\infty}y_jt^j\Big)\Big(\sum_{k=0}^{\infty}v_kt^k\Big)=\sum_{s=0}^{\infty}\sum_{i+j+k=s}\theta(x_i,y_j)v_kt^s.$$

Definition 5.6 *Let T be a relative Rota-Baxter operator on a Hom-Lie-Yamaguti algebra $(L,[\cdot,\cdot],\{\cdot,\cdot,\cdot\},\alpha)$ with respect to the representation $(V;\beta,\rho,D,\theta)$. If $T_t=\sum_{i=0}^{\infty}I_it^i$, $I_i\in\mathrm{Hom}(V,L)$, where $I_0=T$, satisfies*

$$T_t(\beta(u))=\alpha(T_t(u)),\tag{5.8}$$
$$[T_tu,T_tv]=T_t(\rho(T_tu)v-\rho(T_tv)u),\tag{5.9}$$
$$\{T_tu,T_tv,T_tw\}=T_t(D(T_tu,T_tv)w+\theta(T_tv,T_tw)u-\theta(T_tu,T_tw)v),\tag{5.10}$$

for all $u,v,w\in V$. We say that T_t is a formal deformation of T.

For $n\geq 0$, if we compare the coefficients of t^n from both sides of Eqs. (5.8)–(5.10), then we obtain the following system of equations:

$$I_n(\beta(u))=\alpha(I_n(u)),\tag{5.11}$$
$$\sum_{i+j=n}[I_iu,I_jv]-I_i(\rho(I_ju)v-\rho(I_jv)u)=0,\tag{5.12}$$

$$\sum_{i+j+k=n} \{I_i u, I_j v, I_k w\} - I_i(D(I_j u, I_k v)w + \theta(I_j v, I_k w)u - \theta(I_j u, I_k w)v) = 0, \tag{5.13}$$

for all $u, v, w \in V$.

Proposition 5.7 *Let $T_t = \sum_{i=0}^{\infty} I_i t^i$ be a formal deformation of a relative Rota-Baxter operator T on a Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to the representation $(V; \beta, \rho, D, \theta)$. Then $I_1 \in C_T^1(V, L)$ is a (0,1)-cocycle of the relative Rota-Baxter operator T.*

Proof When $n = 1$, by (5.11), we have $I_1 \in C_T^1(V, L)$, and Eqs (5.12) and (5.13) are equivalent to

$$\begin{aligned}
&[Tu, I_1 v] + [I_1 u, Tv] - T(\rho(I_1 u)v - \rho(I_1 v)u) - I_1(\rho(Tu)v - \rho(Tv)u) = 0,\\
&\{I_1 u, Tv, Tw\} + \{Tu, I_1 v, Tw\} + \{Tu, Tv, I_1 w\}\\
&\quad = I_1(D(Tu, Tv)w + \theta(Tv, Tw)u - \theta(Tu, Tw)v) + T(D(I_1 u, Tv)w + \theta(I_1 v, Tw)u -\\
&\qquad \theta(I_1 u, Tw)v) + T(D(Tu, I_1 v)w + \theta(Tv, I_1 w)u - \theta(Tu, I_1 w)v).
\end{aligned}$$

Therefore, Eqs. (4.3) and (4.4) are satisfied, i.e., I_1 is a (0,1)-cocycle of the relative Rota-Baxter operator T. □

Definition 5.8 *Let T be a relative Rota-Baxter operator on a Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to the representation $(V; \beta, \rho, D, \theta)$. Then the $(0,1)$-cocycle I_1 is called the infinitesimal of the formal deformation T_t of T.*

Definition 5.9 *Let T be a relative Rota-Baxter operator on an involutive Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to the representation $(V; \beta, \rho, D, \theta)$. Two formal deformations $T'_t = \sum_{i=0}^{\infty} I'_i t^i$ and $T_t = \sum_{i=0}^{\infty} I_i t^i$ are said to be equivalent if there exist two elements $a, b \in L$ satisfying $\alpha(a) = a$ and $\alpha(b) = b$ such that two linear maps*

$$\phi_t = Id_L + t\mathcal{L}(a, b) + \sum_{i=2}^{\infty} \phi_i t^i, \quad \phi_i \in \mathrm{End}(L), \tag{5.14}$$

$$\varphi_t = Id_V + tD(a, b) + \sum_{i=2}^{\infty} \varphi_i t^i, \quad \varphi_i \in \mathrm{End}(V), \tag{5.15}$$

meet the following equations:

$$\begin{aligned}
&\phi_t[x, y] = [\phi_t(x), \phi_t(y)], \quad \phi_t\{x, y, z\} = \{\phi_t(x), \phi_t(y), \phi_t(z)\},\\
&\varphi_t(D(x, y)v) = D(\phi_t(x), \phi_t(y))\varphi_t(v), \ \ \varphi_t(\theta(x, y)v) = \theta(\phi_t(x), \phi_t(y))\varphi_t(v),\\
&\beta(\varphi_t(v)) = \varphi_t(\beta(v)), \ \ \alpha(\phi_t(x)) = \phi_t(\alpha(x)),\\
&\varphi_t(\rho(x)v) = \rho(\phi_t(x))\varphi_t(v), && (5.16)\\
&T_t(\varphi_t(v)) = \phi_t(T'_t(v)), && (5.17)
\end{aligned}$$

for all $x, y, z \in L, v \in V$.

Theorem 5.10 *Let T be a relative Rota-Baxter operator on an involutive Hom-Lie-Yamaguti algebra $(L, [\cdot,\cdot], \{\cdot,\cdot,\cdot\}, \alpha)$ with respect to the representation $(V; \beta, \rho, D, \theta)$. Two formal defor-*

mations $T'_t = \sum_{i=0}^{\infty} I'_i t^i$ and $T_t = \sum_{i=0}^{\infty} I_i t^i$ of T are equivalent, then their infinitesimals are in the same cohomology class.

Proof Let ϕ_t and φ_t be two linear maps defined in Eqs. (5.14) and (5.15) such that two deformations $T'_t = \sum_{i=0}^{\infty} I'_i t^i$ and $T_t = \sum_{i=0}^{\infty} I_i t^i$ are equivalent. By Eqs. (5.16) and (5.17), we have $\beta^2 = Id_V$ and $I'_1(v) - I_1(v) = T(D(a,b)v) - \{a, b, Tv\}$. From Proposition 4.2, we can get $I'_1(v) - I_1(v) = \delta(a,b)v$, which implies that I'_1 and I_1 are in the same cohomology class. □

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